College Algebra - Introduction

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Algebra: Arithmetic with letters.

Note: Arithmetic is trying to lower our grade!

Number (#) Line: "Real" Numbers – one-part numbers

("-")_____ $|_{-2}$ ____ $|_{-1}$ ___ $|_{-1}$ ___ $|_{1/2}$ ____ $|_{-1}$ ___ $|_{\sqrt{2}}$ ___ ("+")

Note:

a.
$$\frac{0}{b \neq 0} = 0$$

b. $\frac{a \neq 0}{0} =$ Undefined
c. $\frac{0}{0} =$ Indeterminant ; will be discussed in Calculus

Types (Sets: (Collections, ...) of Numbers:

- a. Natural Numbers: {1,2,3,4,5,...}
- b. Whole Numbers: {0,1,2,3,4,5,...}
- c. Integers: {...-4,-3,-2,-1,0,1,2,3,4,...}
- d. **Rational (Fractional) Numbers:** $\left\{ \frac{a}{b} \middle| a \& b \text{ are Integers with } b \neq 0 \right\}$

Examples: $\frac{3}{5}, -\frac{7}{2}, \frac{2}{1} = 2, \dots$

Decimal representation repeats: $\frac{7}{11} = 0.636363...$

e. Irrational Numbers: Non fractional numbers on the Number Line Decimal representation does NOT repeat: $\sqrt{2} = 1.414213562...$

Examples: $\pi = 3.14159..., e = 2.7182818284590..., \sqrt{7} = 2.645751311,...$

Expressions: Combinations of operations (+,-, ...), numbers, letters and grouping symbols ; NO "="

Equations: "=" (There are many types of equations; we will study some of them)

- 1. Linear in "x" (unknown, variable) ; only one letter but can be different than "x"
 - a. "x" is the variable, unknown, ...
 - b. Linear implies power one (1): $x = x^{Power} = x^1$
 - c. Goal: Solve for x (Isolate "x"): $x = \frac{x}{1} = \#or\#s$ when appropriate [No "x" on the RHS Right Hand Side]
 - d. Types of Linear Equations
 - i. Conditional (1 solution only)

$$3x-5 = x+4$$
$$2x = 9$$
$$x = \frac{9}{2}$$

_____9/2____

ii. Identity (Infinite solutions)

3x-6=3(x-2)3x-6=3x-60=0

iii. Contradiction (No solutions)

3x-6 = 3(x+2) 3x-6 = 3x+6 $0 = 12 \implies \text{Contradiction (TRASH!)}$

Note: Always draw the graph of the solution!

2. Literal (Letter) Equations – multiple letters

- a. Considering linear now ; Examples: ax + by = c, ax = b, ...
- b. MUST be given the "Letter" to "solve for":"Letter" = combination of other letters and #'s

3. Terms, Factors, and Exponents

a. Terms: Addition (+) and Subtraction (-)

Terms: Move across "="

- i. Change sign ("+" to "-" **OR** "-" to "+")
- ii. Keep position

Example: $2x - 3 = 4 \Rightarrow 2x = 4 + 3$

b. Factors: Multiplication (*) and Division (/)

Factors: Move across "="

- i. Keep sign
- ii. Change position ("Numerator" to "Denominator" Or"Denominator" to "Numerator")

Example:
$$4x = 5 \Rightarrow x = \frac{5}{4}$$

Note: If there is a *term* property, there may or may NOT be a corresponding *factor* property!!

c. **Exponents** (Shorthand for multiplication): BASE^{Exponent (power,...)} where Base & Exponent are numbers

Example: $5^3 = 5*5*5$ (*Simple* Example – much more will be said about exponents)

4. Order of Operations (Exponents, Factors, & Terms):

- a. If grouping symbols ([],(),{},...) exist, preform the operations from the inner to outer grouping symbols (when fractions are encountered, treat the numerator and denominator separately) as follows
- b. Exponents are to be evaluated first
- c. Then, the operations of multiplication and division from left to right: Factors \rightarrow
- d. Finally, the operations of addition and subtraction from left to right: Terms \rightarrow
- 5. TBD To Be Determined ...