Equations – Linear

MATH by Wilson
Your Personal Mathematics Trainer
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Equations: **Equations** have **EQUAL SIGNS**!

IF you want a better understanding of what is going on, then you will "check" all your potential solutions!

Example:

$2\mathbf{x} - 4 = 6 \Rightarrow 2\mathbf{x} = 10 \Rightarrow \mathbf{x} = 5$

Check: $2(5) - 4 \overset{?}{=} 6 \Rightarrow 6 \overset{\text{Yes}}{=} 6$

Without letters – except for the unknown

(1) **Question:** Find the solution of the equation $3 - (4 - 2\mathbf{x}) = 3(\mathbf{x} + 2) - 4\mathbf{x} + 2$

Solution:

Step	Equation	Reason
0	$3 - (4 - 2\mathbf{x}) = 3(\mathbf{x} + 2) - 4\mathbf{x} + 2$	
1	$3 - 4 + 2\mathbf{x} = 3\mathbf{x} + 6 - 4\mathbf{x} + 2$	
2	$-1 + 2\mathbf{x} = -\mathbf{x} + 8$	
3	$\mathbf{x} + 2\mathbf{x} = 8 + 1$	
4	$3\mathbf{x} = 9$	
5	$\mathbf{x} = 3$	

Solution graph: _____|_____3_____
0 |

(2) **Question**: Solve for $\mathbf{x}$ in the equation $2(5\mathbf{x}-3)=7-2\mathbf{x}$

Solution:

Step	Equation	Reason
0	$2(5\mathbf{x}-3)=7-2\mathbf{x}$	
1	$10\mathbf{x}-6=7-2\mathbf{x}$	
2	$2\mathbf{x}+10\mathbf{x}=6+7$	
3	$12\mathbf{x}=13$	
4	$\mathbf{x}=\frac{13}{12}$	

Solution graph: _____|_____13/12_____
0 |

(3) **Question**: Find the solution $\mathbf{x}$ in the equation $\frac{3}{4}\mathbf{x}-2=\frac{1}{3}+2\mathbf{x}$

Solution:

Step	Equation	Reason
0	$\frac{3}{4}\mathbf{x}-2=\frac{1}{3}+2\mathbf{x}$	
1	$12\left(\frac{3\mathbf{x}}{4}-\frac{2}{1}\right)=12\left(\frac{1}{3}+\frac{2\mathbf{x}}{1}\right)$	Eliminate fractions with common denominator
2	$9\mathbf{x}-24=4+24\mathbf{x}$	
3	$9\mathbf{x}-24\mathbf{x}=24+4$	
4	$-15\mathbf{x}=28$	
5	$\mathbf{x}=-\frac{28}{15}$	

Solution graph: _____−28/15_____|_____
| 0

(4) **Question**: Solve for $\mathbf{x}$: $\frac{2}{1-\mathbf{x}} = \frac{4}{3}$

Solution:

Step	Equation	Reason
0	$\frac{2}{1-\mathbf{x}} = \frac{4}{3}$	
1	$2*3 = 4(1-\mathbf{x})$	
2	$6 = 4 - 4\mathbf{x}$	
3	$4\mathbf{x} = 4 - 6$	
4	$4\mathbf{x} = -2$	
5	$\mathbf{x} = -\frac{2}{4} = -\frac{1}{2}$	

Solution graph: _____ $-1/2$ _____ $|_0$ _____

Note: This equation is a linear "rational" equation since the "x" is in the denominator but can be converted to the usual linear format.

With letters – Literal Equations

(5) **Question**: The solution $\mathbf{x}$ of the equation $\frac{\mathbf{a}}{\mathbf{x}}=\frac{\mathbf{b}}{\mathbf{a}}$ is x = ?

Solution:

Step	Equation	Reason
0	$\frac{\mathbf{a}}{\mathbf{x}}=\frac{\mathbf{b}}{\mathbf{a}}$	
1	$\mathbf{a}^2=\mathbf{bx}$	
2	$\frac{\mathbf{a}^2}{\mathbf{b}}=\mathbf{x}$ **OR** $\mathbf{x}=\frac{\mathbf{a}^2}{\mathbf{b}}$	

Note: Can not graph the solution set.

(6) **Question**: The solution $\mathbf{x}$ of the equation $\mathbf{a}-\mathbf{bx}=\mathbf{c}(2-\mathbf{x})$ is x = ?

Solution:

Step	Equation	Reason
0	$\mathbf{a}-\mathbf{bx}=\mathbf{c}(2-\mathbf{x})$	
1	$\mathbf{a}-\mathbf{bx}=2\mathbf{c}-\mathbf{cx}$	
2	$\mathbf{a}-2\mathbf{c}=\mathbf{bx}-\mathbf{cx}$	Group all the x terms together
3	$\mathbf{a}-2\mathbf{c}=\mathbf{x}(\mathbf{b}-\mathbf{c})$	
4	$\frac{\mathbf{a}-2\mathbf{c}}{\mathbf{b}-\mathbf{c}}=\mathbf{x}$ **OR** $\mathbf{x}=\frac{\mathbf{a}-2\mathbf{c}}{\mathbf{b}-\mathbf{c}}=\frac{2\mathbf{c}-\mathbf{a}}{\mathbf{c}-\mathbf{b}}$	

(7) **Question**: Solve the following equations:

a. $2\mathbf{x}=3\Rightarrow\mathbf{x}=\frac{3}{2}$

b. $-4\mathbf{x}=7\Rightarrow\mathbf{x}=-\frac{7}{4}$

c. … There are an infinite number of equations like these. However, consider $\mathbf{ax}=\mathbf{b}$; $\mathbf{a}\neq 0$. We have

$$\mathbf{ax}=\mathbf{b}\Rightarrow\mathbf{x}=\frac{\mathbf{b}}{\mathbf{a}}$$

Note: We have actually solved an infinite number of equations. This is the power of algebra!

