

Equations – Quadratic: Introduction

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Your Personal Mathematics Trainer
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Goal: Solve Quadratic Equations.

Standard Form:

$$ax^2 + bx + c = 0$$

where $a \neq 0, b, c$ are real numbers

Discriminate: $D = b^2 - 4ac$ gives us information about the solutions:

Quadratic Equations have two (2) solutions:

1. Real Numbers:

- a. Same number twice: $D = b^2 - 4ac = 0$
- b. Different numbers: $D = b^2 - 4ac > 0$

2. Complex Numbers – must occur in conjugate pairs:

$$D = b^2 - 4ac < 0$$

Depending upon the values of a,b,c there are different techniques to solve the equation:

1. If $b = 0$: Square Root Technique

$$ax^2 + c = 0$$

$$x^2 = -\frac{c}{a}$$

$$x = \pm \sqrt{-\frac{c}{a}}$$

2. If $c = 0$: Factor Technique

$$ax^2 + bx = 0$$

$$x(ax + b) = 0$$

$$x = 0 \quad | \quad x = -\frac{b}{a}$$

3. If $a \neq 0; b \neq 0; c \neq 0$: **Three Techniques**

- a. **Factor** and set each factor equal to zero
- b. **Complete the Square** & then use the Square Root Technique
- c. Use the **Quadratic Formula (QF)**: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Help from the past:

1. Zero Product Fact:

$$\square * \triangle = 0 \Rightarrow \square = 0 \text{ or } \triangle = 0 \text{ or both equal } 0$$

Example:

$$x^2 + 2x - 15 = 0 \Rightarrow (x - 3)(x + 5) = 0$$

$$\Rightarrow x - 3 = 0 ; x + 5 = 0$$

$$\Rightarrow x = -5, 3$$

2. Factor Help:

a. $(x + b)(x + d) = x^2 + (b + d)x + bd$

Example:

$$x^2 + 2x - 15 = 0 \Rightarrow bd = -15 \text{ \& } b + d \text{ MUST be } 2$$

b	d	b+d
15	-1	14
-15	1	-14
3	-5	-2
-3	5	2 (YES!)

$$\therefore x^2 + 2x - 15 = 0 \Rightarrow (x - 3)(x + 5) = 0$$

b. $(ax + b)(cx + d) = acx^2 + (bc + ad)x + bd$

Example:

$$8x^2 + 2x - 15 = 0 \Rightarrow ac = 8, bd = -5 \text{ \& } bc + ad \text{ MUST be 2}$$

a	c	b	d	bc + ad
2	4	-15	1	-58
2	4	15	-1	58
2	4	-3	5	-2
2	4	3	-5	2 (YES!)

$$\therefore 8x^2 + 2x - 15 = 0 \Rightarrow (2x + 3)(4x - 5) = 0$$

3. Complete the Square Help:

Perfect Square:

$$\begin{aligned} (x + a)^2 &= x^2 + 2ax + a^2 \\ &= 1 * x^2 + 2ax + a^2 \\ &= [x^2 - \text{coefficient}] * x^2 + [x - \text{coefficient}] * x + [\text{constant}] \end{aligned}$$

$$\therefore x^2 - \text{coefficient} = 1 ; x - \text{coefficient} = 2a ; \text{constant} = a^2$$

Important:

1. $x^2 - \text{coefficient} = 1$

2. $x - \text{coefficient} = 2a$

$$\frac{1}{2}(x - \text{coefficient}) = a$$

$$\left[\frac{1}{2}(x - \text{coefficient}) \right]^2 = a^2 = \text{constant}$$

Example:

Steps	Example
$x^2 + 2ax = \#$	$x^2 + 6x - 7 = 0 \Rightarrow x^2 + 6x = 7$
$x^2 + 2ax + \left[\frac{1}{2}(x - \text{coeff}) \right]^2$ $= \# + \left[\frac{1}{2}(x - \text{coeff}) \right]^2$	$x^2 + 6x + \left(\frac{6}{2} \right)^2 = 7 + \left(\frac{6}{2} \right)^2$
$x^2 + 2ax + a^2 = \# + a^2$	$x^2 + 6x + 9 = 7 + 9$
$(x + a)^2 = \# + a^2$	$(x + 3)^2 = 16$
$x + a = \pm \sqrt{\# + a^2}$	$x + 3 = \pm \sqrt{16} = \pm 4$ (2 Solutions!)
$x = -a \pm \sqrt{\# + a^2}$	$x = -3 \pm 4 \Rightarrow x = -7, 1$