

# Equations – Quadratic: Solve

MATH by Wilson  
Your Personal Mathematics Trainer  
MathByWilson.com

**Goal:** Solve various Quadratic Equations

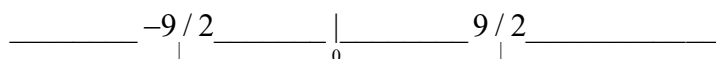
(1) **Question:** Solve the equation  $4x^2 - 81 = 0$ .

**Solution:**

Note:  $a = 4$  ;  $b = 0$  ;  $c = -81$

Step	Equation	Reason
0	$4x^2 - 81 = 0$	
1	$4x^2 = 81$	
2	$x^2 = \frac{81}{4}$	
3	$x = \pm \sqrt{\frac{81}{4}} = \pm \frac{9}{2}$	

Graph of solution set:



**Note:** there are certainly other ways to solve this and other equations.

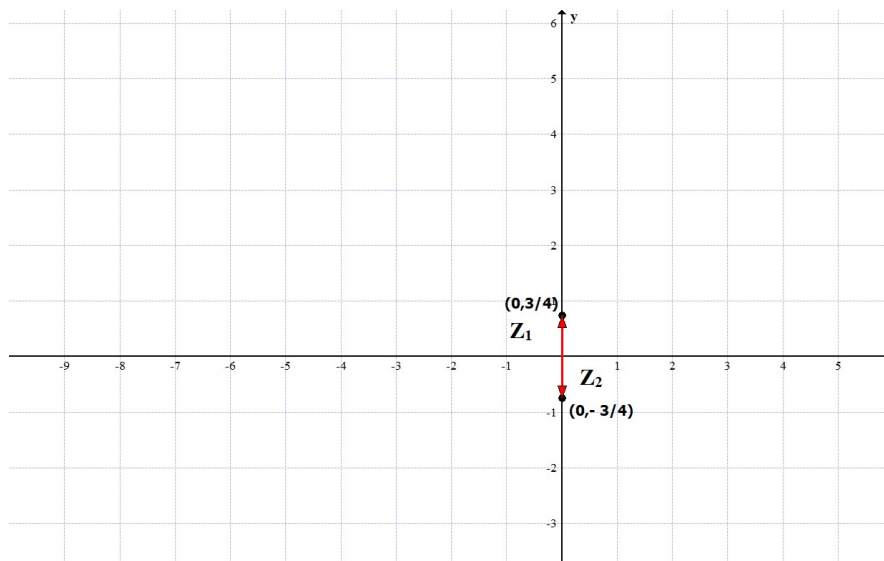
(2) **Question:** The solution(s) of the equation  $\frac{x^2}{9} + \frac{1}{16} = 0$  satisfies

- A) One real solution (“root of multiplicity two”)
- B) Two real solutions (“distinct solutions”)
- C) Two complex solutions (NOT conjugate pairs)
- D) Two complex solutions (Conjugate pairs)

**Solution:**

Step	Equation	Reason
0	$\frac{x^2}{9} + \frac{1}{16} = 0$	
1	$\frac{x^2}{9} = -\frac{1}{16}$	
2	$x^2 = -\frac{9}{16}$	
3	$x = \pm \sqrt{-\frac{9}{16}}$	
4	$x = \pm \sqrt{\frac{9}{16}} * \sqrt{-1}$	
5	$x = \pm \frac{3}{4} i$	

Graph of the solution set:



(3) **Question:** Find the *smallest* solution of the equation  $x^2 - 14x + 48 = 0$ .

**Solution:**

Note:  $a = 1$ ;  $b = -14$ ;  $c = 48$

Three (3) solutions are given below:

**Factor:**

Step	Equation	Reason
0	$x^2 - 14x + 48 = 0$	
1	$(x - 6)(x - 8) = 0$	
2	$x - 6 = 0$    $x - 8 = 0$ $x = 6$    $x = 8$	

**Complete the Square:**

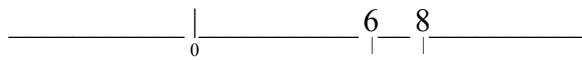
Step	Equation	Reason
0	$x^2 - 14x + 48 = 0$	
1	$x^2 - 14x = -48$	x-coeff: -14 1/2 (x-coeff): -7 [1/2(x-coeff)] <sup>2</sup> : 49
2	$x^2 - 14x + 49 = -48 + 49$	
3	$(x - 7)^2 = 1$	
4	$x - 7 = \pm 1$	
5	$x - 7 = -1$    $x - 7 = 1$ $x = 6$    $x = 8$	

**Quadratic Formula - QF:**

Step	Equation	Reason
0	$x^2 - 14x + 48 = 0$	
1	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	
2	$x = \frac{-[-14] \pm \sqrt{[-14]^2 - 4[1][48]}}{2[1]}$	

3	$x = \frac{14 \pm \sqrt{196 - 192}}{2}$	
4	$x = \frac{14 \pm \sqrt{4}}{2}$	
5	$x = \frac{14 \pm 2}{2}$	
6	$\begin{array}{l} x = \frac{14-2}{2} \\ x = 6 \end{array} \parallel \begin{array}{l} x = \frac{14+2}{2} \\ x = 8 \end{array}$	

Graph of the solution set:



(4) **Question:** Find the *sum* of the solutions of the equation  $10x^2 - 11x = 6$ .

**Solution:**

**Standard Form:**  $10x^2 - 11x - 6 = 0$  so that  $\mathbf{a} = 10$  ;  $\mathbf{b} = -11$  ;  $\mathbf{c} = -6$

Again, three (s) solutions:

**Factor:**

Step	Equation	Reason
0	$10x^2 - 11x - 6 = 0$	
1	$(5x+2)(2x-3) = 0$	
2	$\begin{array}{l} 5x+2=0 \\ 5x=-2 \\ x=-\frac{2}{5} \end{array} \parallel \begin{array}{l} 2x-3=0 \\ 2x=3 \\ x=\frac{3}{2} \end{array}$	

### Complete the Square:

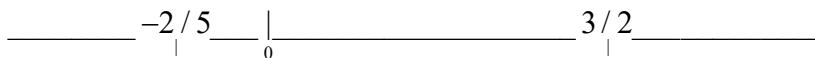
Step	Equation	Reason
0	$10x^2 - 11x - 6 = 0$	
1	$10\left[x^2 - \frac{11}{10}x\right] = 6$	x-coeff: $-\frac{11}{10}$ 1/2 (x-coeff): $-\frac{11}{20}$ [1/2(x-coeff)] <sup>2</sup> : $\frac{121}{400}$
2	$10\left[x^2 - \frac{11}{10}x + \frac{121}{400}\right] = 6 + 10 * \frac{121}{400}$	
3	$10\left[x - \frac{11}{20}\right]^2 = \frac{361}{40}$	
4	$\left[x - \frac{11}{20}\right]^2 = \frac{361}{400}$	
5	$x - \frac{11}{20} = \pm \sqrt{\frac{361}{400}} = \pm \frac{19}{20}$	
6	$\begin{array}{l} x - \frac{11}{20} = -\frac{19}{20} \\ x = \frac{11}{20} - \frac{19}{20} \\ x = -\frac{8}{20} = -\frac{2}{5} \end{array} \quad \left\  \quad \begin{array}{l} x - \frac{11}{20} = \frac{19}{20} \\ x = \frac{11}{20} + \frac{19}{20} \\ x = \frac{30}{20} = \frac{3}{2} \end{array} \right.$	

### Quadratic Formula - QF:

Step	Equation	Reason
0	$10x^2 - 11x - 6 = 0$	
1	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	
2	$x = \frac{-[-11] \pm \sqrt{[-11]^2 - 4[10][-6]}}{2[10]}$	
3	$x = \frac{11 \pm \sqrt{121 + 240}}{20}$	
4	$x = \frac{11 \pm \sqrt{361}}{20}$	

5	$x = \frac{11 \pm 19}{20}$	
6	$x = \frac{11-19}{20}$ $x = \frac{11+19}{20}$ $x = -\frac{5}{2}$ $x = \frac{3}{2}$	

Graph of the solution set:



(5) **Question:** Find the *largest* solution  $x_{\text{Large}}$  of the equation  $3x^2 + 2x - 4 = 0$ .

**Solution:**

Note:  $a = 3$  ;  $b = 2$  ;  $c = -4$

Only two (2) solutions:

**Factor:**

Doesn't factor "nicely"! **UGLY!**

**Complete the Square:**

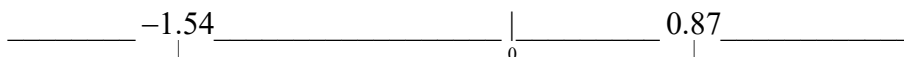
Step	Equation	Reason
0	$3x^2 + 2x - 4 = 0$	
1	$3 \left[ x^2 + \frac{2}{3}x \right] = 4$	$x\text{-coeff: } \frac{2}{3}$ $1/2 (x\text{-coeff}): \frac{1}{3}$ $[1/2(x\text{-coeff})]^2: \frac{1}{9}$
2	$3 \left[ x^2 + \frac{2}{3}x + \frac{1}{9} \right] = 4 + 3 * \frac{1}{9}$	

3	$3\left[x + \frac{1}{3}\right]^2 = \frac{13}{3}$	
4	$\left[x + \frac{1}{3}\right]^2 = \frac{13}{9}$	
5	$x + \frac{1}{3} = \pm \sqrt{\frac{13}{9}} = \pm \frac{\sqrt{13}}{3}$	
6	$\begin{array}{l} x + \frac{1}{3} = -\frac{\sqrt{13}}{3} \\ x = -\frac{1 + \sqrt{13}}{3} \\ x \approx -1.5352 \end{array} \quad \parallel \quad \begin{array}{l} x + \frac{1}{3} = \frac{\sqrt{13}}{3} \\ x = \frac{-1 + \sqrt{13}}{3} \\ x \approx 0.8685 \end{array}$	

### Quadratic Formula - QF:

Step	Equation	Reason
0	$3x^2 + 2x - 4 = 0$	
1	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	
2	$x = \frac{-[-2] \pm \sqrt{[-2]^2 - 4[3][-4]}}{2[3]}$	
3	$x = \frac{2 \pm \sqrt{4 + 48}}{6}$	
4	$x = \frac{2 \pm \sqrt{52}}{6} = \frac{2 \pm 2\sqrt{13}}{6}$	
5	$x = \frac{1 \pm \sqrt{13}}{3}$	
6	$\begin{array}{l} x = \frac{1 - \sqrt{13}}{3} \\ x \approx -1.5352 \end{array} \quad \parallel \quad \begin{array}{l} x = \frac{1 + \sqrt{13}}{3} \\ x \approx 0.8685 \end{array}$	

Graph of the solution set:



(6) **Question:** Is there a *largest* solution  $x_{\text{Large}}$  of the equation  $x^2 + 3x + 5 = 0$ ?

**Solution:**

Note:  $a = 1$  ;  $b = 3$  ;  $c = 5$

Ugly again!

**Factor:**

Doesn't factor "nicely"! **UGLY!**

**Complete the Square:**

Step	Equation	Reason
0	$x^2 + 3x + 5 = 0$	
1	$x^2 + 3x = -5$	x-coeff: 3 1/2 (x-coeff): $\frac{3}{2}$ [1/2(x-coeff)] <sup>2</sup> : $\frac{9}{4}$
2	$x^2 + 3x + \frac{9}{4} = -5 + \frac{9}{4}$	
3	$\left(x + \frac{3}{2}\right)^2 = -\frac{11}{4}$	
4	$x + \frac{3}{2} = \pm \sqrt{-\frac{11}{4}}$	
5	$x + \frac{3}{2} = \pm \frac{\sqrt{-11}}{2} = \pm \frac{\sqrt{11} i}{2}$	
6	$x + \frac{3}{2} = -\frac{\sqrt{11} i}{2} \quad \left\  \quad x + \frac{3}{2} = \frac{\sqrt{11} i}{2}\right.$ $x = -\frac{3 + \sqrt{11} i}{2} \quad \left\  \quad x = \frac{-3 + \sqrt{11} i}{2}\right.$	



## Quadratic Formula - QF:

Step	Equation	Reason
0	$x^2 + 3x + 5 = 0$	
1	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	
2	$x = \frac{-[3] \pm \sqrt{[3]^2 - 4[1][5]}}{2[1]}$	
3	$x = \frac{-3 \pm \sqrt{9 - 20}}{2}$	
4	$x = \frac{-3 \pm \sqrt{-11}}{2}$	
5	$x = \frac{-3 \pm \sqrt{11} i}{2}$	
6	$x = \frac{-3 - \sqrt{11} i}{2} \parallel x = \frac{-3 + \sqrt{11} i}{2}$	

Graph of the solution set:

