Equations – Quadratic in Form

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Your Personal Mathematics Trainer
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Quadratic Equation: $\mathbf{au^2 + bu + c = 0}$ (Standard Form) in **u** (called the universal variable)

The **solutions** are $\mathbf{u} = \dfrac{-\mathbf{b} \pm \sqrt{\mathbf{b}^2 - 4\mathbf{ac}}}{2\mathbf{a}}$ using the Quadratic Equation.

***** **u** can be any of the following plus plenty of others in the original equation.

$\mathbf{u} = \mathbf{x}$

$\mathbf{u} = \dfrac{1}{\mathbf{x}^2}$

$\mathbf{u} = \sqrt[3]{\mathbf{x}} = \mathbf{x}^{1/3}$

$\mathbf{u} = \mathbf{x}^2$

$\mathbf{u} = \dfrac{1}{\mathbf{x}}$

$\mathbf{u} = \mathbf{x} - 2$

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Strategy: In the original equation, we

1. set u = "something" so we get a quadratic equation in u: $\mathbf{au^2 + bu + c = 0}$
2. solve this equation for **u**
3. use the solutions for **u** to get the original solutions for **x**

We'll look at equations – Quadratic in Form – to get the following equations in **u**:

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$\mathbf{u}^2 - 14\mathbf{u} + 45 = 0$

$\mathbf{u}^2 + 7\mathbf{u} + 12 = 0$

$\mathbf{u}^2 - 13\mathbf{u} + 36 = 0$

$6\mathbf{u}^2 - 11\mathbf{u} - 10 = 0$

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Question 01: Find the solutions of the equation $\frac{1}{\mathbf{x}^4}-14\frac{1}{\mathbf{x}^2}+45=0$.

Solution:

Set $\mathbf{u}=\frac{1}{\mathbf{x}^2}$ so that

$$\frac{1}{\mathbf{x}^4}-14\frac{1}{\mathbf{x}^2}+45=0$$
$$\left(\frac{1}{\mathbf{x}^2}\right)^2-14\left(\frac{1}{\mathbf{x}^2}\right)+45=0$$
$$\mathbf{u}^2-14\mathbf{u}+45=0$$

Note: $\mathbf{a}=1$; $\mathbf{b}=-14$; $\mathbf{c}=45$

Step	Equation	Reason
0	$\mathbf{u}^2-14\mathbf{u}+45=0$	
1	$(\mathbf{u}-9)(\mathbf{u}-5)=0$	
2	$\mathbf{u}-9=0 \quad \Vert \quad \mathbf{u}-5=0$ $\mathbf{u}=9 \quad \Vert \quad \mathbf{u}=5$	
3	$\frac{1}{\mathbf{x}^2}=9 \quad \Vert \quad \frac{1}{\mathbf{x}^2}=5$ $\mathbf{x}^2=\frac{1}{9} \quad \Vert \quad \mathbf{x}^2=\frac{1}{5}$ $\mathbf{x}=\pm\frac{1}{3} \quad \Vert \quad \mathbf{x}=\pm\frac{1}{\sqrt{5}}=\pm\frac{\sqrt{5}}{5}$	

Graph of the solution set:

$$\underset{}{\_\_\_\_\_}\ \underset{|}{-1/\sqrt{5}}\ \underset{|}{-1/3}\ \underset{0}{\_\_\_\_\_|\_\_\_\_\_}\ \underset{|}{1/3}\ \underset{|}{1/\sqrt{5}}\ \underset{}{\_\_\_\_\_}$$

Question 02: The equation $\mathbf{x}^{2/3}+7\mathbf{x}^{1/3}+12=0$ has how many solutions?
Solution:

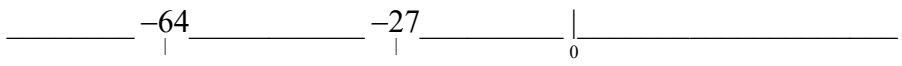
Set $\mathbf{u}=\mathbf{x}^{1/3}$ so that

$$\mathbf{x}^{2/3}+7\mathbf{x}^{1/3}+12=0$$
$$\left(\mathbf{x}^{1/3}\right)^2+7\left(\mathbf{x}^{1/3}\right)+12=0$$
$$\mathbf{u}^2+7\mathbf{u}+12=0$$

Note: $\mathbf{a}=1$; $\mathbf{b}=7$; $\mathbf{c}=12$

Step	Equation	Reason
0	$\mathbf{u}^2+7\mathbf{u}+12=0$	
1	$(\mathbf{u}+4)(\mathbf{u}+3)=0$	
2	$\mathbf{u}+4=0$ || $\mathbf{u}+3=0$ $\mathbf{u}=-4$ || $\mathbf{u}=-3$	
3	$\mathbf{x}^{1/3}=-4$ $\mathbf{x}=\left(\mathbf{x}^{1/3}\right)^3=(-4)^3$ $\mathbf{x}=-64$ || $\mathbf{x}^{1/3}=-3$ $\mathbf{x}=\left(\mathbf{x}^{1/3}\right)^3=(-3)^3$ $\mathbf{x}=-27$	

Grapy of the solution set:



Question 03: Find the *largest* solution of the equation $\mathbf{x}^4-13\mathbf{x}^2+36=0$.
Solution:

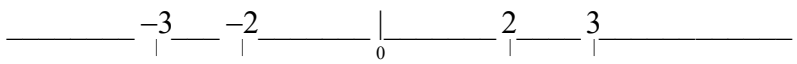
Set $\mathbf{u}=\mathbf{x}^2$ so that

$$\mathbf{x}^4-13\mathbf{x}^2+36=0$$
$$\left(\mathbf{x}^2\right)^2-13\left(\mathbf{x}^2\right)+36=0$$
$$\mathbf{u}^2-13\mathbf{u}+36=0$$

Note: $\mathbf{a}=1$; $\mathbf{b}=-13$; $\mathbf{c}=36$

Step	Equation	Reason
0	$\mathbf{u}^2 - 13\mathbf{u} + 36 = 0$	
1	$(\mathbf{u} - 4)(\mathbf{u} - 9) = 0$	
2	$\mathbf{u} - 4 = 0 \quad \Vert \quad \mathbf{u} - 9 = 0$ $\mathbf{u} = 4 \quad \Vert \quad \mathbf{u} = 9$	
3	$\mathbf{x}^2 = 4 \quad \Vert \quad \mathbf{x}^2 = 9$ $\mathbf{x} = \pm\sqrt{4} = \pm 2 \quad \Vert \quad \mathbf{x} = \pm\sqrt{9} = \pm 3$	

Graph of the solution set:



Question 04: What is the **sum** of the solutions of the equation

$$\frac{6}{\mathbf{x}^2} - 11\frac{1}{\mathbf{x}} - 10 = 0 \text{ ?}$$

Solution:

Set $\mathbf{u} = \frac{1}{\mathbf{x}}$ so that

$$\frac{6}{\mathbf{x}^2} - 11\frac{1}{\mathbf{x}} - 10 = 0$$

$$6\left(\frac{1}{\mathbf{x}}\right)^2 - 11\left(\frac{1}{\mathbf{x}}\right) - 10 = 0$$

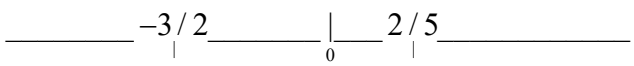
$$6\mathbf{u}^2 - 11\mathbf{u} - 10 = 0$$

Note: $\mathbf{a} = 6$; $\mathbf{b} = -11$; $\mathbf{c} = -10$

Step	Equation	Reason
0	$6\mathbf{u}^2 - 11\mathbf{u} - 10 = 0$	
1	$(3\mathbf{u} + 2)(2\mathbf{u} - 5) = 0$	
2	$3\mathbf{u} + 2 = 0 \quad \Vert \quad 2\mathbf{u} - 5 = 0$ $3\mathbf{u} = -2 \quad \Vert \quad 2\mathbf{u} = 5$ $\mathbf{u} = -\frac{2}{3} \quad \Vert \quad \mathbf{u} = \frac{5}{2}$	

3	$\frac{1}{\mathbf{x}} = -\frac{2}{3}$ $\mathbf{x} = -\frac{3}{2}$ $\frac{1}{\mathbf{x}} = \frac{5}{2}$ $\mathbf{x} = \frac{2}{5}$	

Graph of the solution set:



Sum of solutions is -11/10