

Equations – Radical [Roots]

[MATH by Wilson
Your Personal Mathematics Trainer
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The **basic idea** is to trade an equation with one or more radicals - $(\sqrt[n]{\text{Expression}})$ - for an equation that we have already studied that does NOT have radicals:

Original Equation: Contains Radicals

TRADE

New Equation: Does NOT Contain Radicals

Note that $(\sqrt[n]{\text{Expression}})^n = [(\text{Expression})^{1/n}]^n = \text{Expression}$ is key to our success!

Warning: The theory tells me to tell you that a solution of the New Equation does NOT have to be a solution of the Original Equation. Therefore, it is **mandatory** to check “potential” solutions to ensure that they are “actual” solutions.

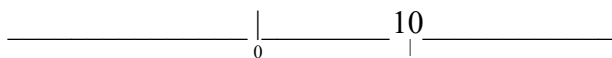
Note: We should *always* graph our solutions, if any, on the number line.

Question 01: Solve for x: $\sqrt{2x+5} = \frac{x}{2}$

Solution:

Step	Equation	Reason
0	$\sqrt{2x+5} = \frac{x}{2}$	
1	$(\sqrt{2x+5})^2 = \left(\frac{x}{2}\right)^2$	Eliminate Radical
2	$2x+5 = \frac{x^2}{4}$	
3	$8x+20 = x^2$	
4	$x^2 - 8x - 20 = 0$	
5	$(x+2)(x-10) = 0$	
6	$x+2=0 \quad \quad x-10=0$ $x=-2 \quad \quad x=10$	
7	$x = -2$: Is NOT a solution $\sqrt{2[-2]+5} \stackrel{?}{=} \frac{[-2]}{2}$ $1 \neq -1$	
8	$x = 10$: Is a solution $\sqrt{2[10]+5} \stackrel{?}{=} \frac{[10]}{2}$ $5 = 5$	

Graph of the solution set:

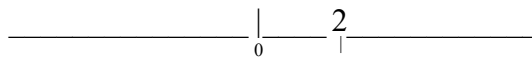


Question 02: Solve for x: $\sqrt{6-x} + \sqrt{x+2} = 4$ (This is a long problem because there are two radicals to eliminate **and** they must be eliminated correctly!

Solution:

Step	Equation	Reason
0	$\sqrt{6-x} + \sqrt{x+2} = 4$	
1	$\sqrt{6-x} = 4 - \sqrt{x+2}$	
2	$(\sqrt{6-x})^2 = (4 - \sqrt{x+2})^2$	Eliminate 1 st Radical
3	$6 - x = 16 - 8\sqrt{x+2} + (x+2)$	
4	$6 - x = x + 18 - 8\sqrt{x+2}$	
5	$-2x - 12 = -8\sqrt{x+2}$	
6	$x + 6 = 4\sqrt{x+2}$	Divide by “-2”
7	$(x+6)^2 = (4\sqrt{x+2})^2$	Eliminate 2 nd Radical
8	$x^2 + 12x + 36 = 16(x+2) = 16x + 32$	
9	$x^2 - 4x + 4 = 0$	
10	$(x-2)^2 = 0$	
11	$x - 2 = 0 \quad \quad x - 2 = 0$ $x = 2 \quad \quad x = 2$	
12	$x = 2$: Is a solution $\sqrt{6 - [2]} + \sqrt{[2] + 2} = 4$ $2 + 2 = 4$ $4 = 4$	

Graph of the solution set:



Equation 03: Solve for x: $\sqrt[3]{3-x} = 3$

Solution:

Step	Equation	Reason
0	$\sqrt[3]{3-x} = 3$	
1	$(\sqrt[3]{3-x})^3 = (3)^3$	Eliminate Radical
2	$3-x = 27$	
3	$-x = 24 \Rightarrow x = -24$	
7	$x = -24$: Is a solution $\sqrt[3]{3-[-24]} = 3$ $\sqrt[3]{27} = 3$ $3 = 3$	

Graph of the solution set:

