Inequalities – Introduction

Equality (Equivalence) & Inequality (Non-Equality)

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Your Personal Mathematics Trainer
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1. Equality

The word equality comes from the word equal which states that two (2) items are the same. We use the sign = to denote equality (& equivalence):

Right Hand Side = Left and Side [RHS = LHS]

Denote **equality**

$$3 = 3$$

or

equivalence

$$3 = \frac{3}{1}$$
$$3 = \frac{12}{4}$$
$$3 = \sqrt{9}$$
$$3 = \sqrt[3]{27}$$
$$3 = |-3|$$
$$3 = 3.00$$

.
.
.

Note: With an equivalence, the values are the same but the forms are different. Different forms are used for different purposes.

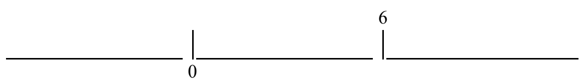
Frequently in Mathematics, we are asked to **solve** an equality (called an **equation**) for a letter, say "x", which means that we are looking for a number or numbers, that make(s) the equation true:

$$2(\mathbf{x}-4)=13-\mathbf{x}$$
$$2([?]-4)=13-[?]$$

The solution is $\mathbf{x}=7$ since

$$2([7]-4)=13-[7]$$
$$6=6$$

Graph of the solution set:



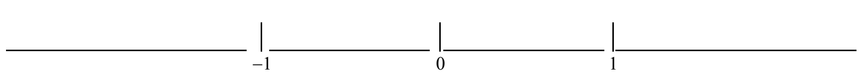
Note that sometimes in Mathematics, we must change the form but not the value in order "to do the math":

$$\begin{aligned}\frac{1}{6}+\frac{8}{3}&=\frac{1}{6}+\frac{8}{3}*1\\&=\frac{1}{6}+\frac{8}{3}*\frac{2}{2}\\&=\frac{1}{6}+\frac{16}{6}\\&=\frac{17}{6}\end{aligned}$$

FYI: The above "form changer" comes from the identities $\frac{a}{a}=1\,;\,a\neq 0$ & $a=a*1$. Another "form changer" is $a=a+0$

2. Inequality

If two (2) items are NOT equal or equivalent, we use the symbol $\neq$ **.**
Consider two (2) numbers a and b on the number line (think fancy "ruler"):



Number Line

If they are equal (a = b), they reside at the same location on the number line:

__________________________ | __________________________ Number Line
a = b

If not, we write **a ≠ b** and one number must be to the left of the other one:

________________ |_____________________ |________________ Number Line
a b

In this case, we write

a < b (or b > a)

and say that "a" is *less than* "b" (or "b" is *greater than* "a") If "b" is to the left of "a",

________________ |_____________________ |________________ Number Line
b a

we write b < a (or a > b) and say that "a" is *greater than* "b" (or "b" is *less than* "a"). Hence,

$3 < 5$ (or $5 > 3$)
$7 > 4$ (or $4 < 7$)

Writing x < 6 means that "x" represents *any* number less than 6:

/)________________
6
Note: ")" means excluded

For example

$-62 < 6\ ;\ -\frac{3}{4} < 6\ ;\ 0 < 6\ ;\ 2\frac{1}{2} < 6$

The statement 11 < 6 is FALSE which we write as $11 \not< 6$.

Writing -2 < x means that "x" represents *any* number greater than -2:

$$\underline{\qquad\qquad\qquad}\underset{-2}{(}//////////////////////////$$

For example

$$-2<-1\frac{3}{5}\ ;\ -2<0\ ;\ -2<3\frac{2}{7}\ ;\ -2<73$$

Sometimes we want to allow equality so we write $\mathbf{a}\le\mathbf{b}$ (or $\mathbf{b}\ge\mathbf{a}$) which means a < b **OR** a = b:

$$\overbrace{////////////////////}^{\mathbf{a}}\underset{\mathbf{b}}{]}\underline{\qquad\qquad\qquad}$$

Note: "]" means inclusion

We write $\mathbf{a}\ge\mathbf{b}$ (or $\mathbf{b}\le\mathbf{a}$) which means a > b **OR** a = b:

$$\underline{\qquad\qquad\qquad}\underset{\mathbf{b}}{[}\overbrace{////////////////////}^{\mathbf{a}}$$

So

$7\le 13$

since

$7<13$

and

$7\le 7$

since

$7=7$

