Inequalities
Quadratic

$$\begin{bmatrix} \text{MATH by Wilson} \\ \text{Your Personal Mathematics Trainer} \\ \text{MathByWilson.com} \end{bmatrix}$$
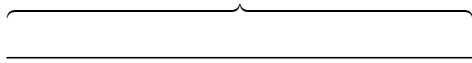
Procedure (Test Point Method):

1. **With the quadratic equation in standard form**

$$\mathbf{ax^2 + bx + c} \begin{Bmatrix} < \\ \leq \\ > \\ \geq \end{Bmatrix} \mathbf{0}$$

   find the solutions x_1 ; x_2 of $ax^2 + bx + c = 0$. The solutions divide the horizontal number line into regions:
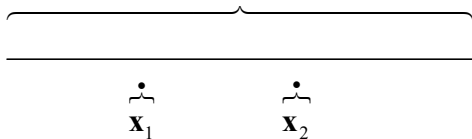
   a. **One (1) region if the solutions are complex numbers.**

   b. **Two (2) regions if the solutions are the same real number, that is $x_1 = x_2$.**
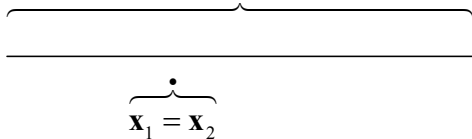
$$\overset{\bullet}{x_1 = x_2}$$

   c. **Three (3) regions if the solutions differ, that is, $x_1 \neq x_2$**

$$\overset{\bullet}{x_1} \qquad \overset{\bullet}{x_2}$$

2. **If x_1 or x_2 satisfies the original inequality, the numbers are part of the solution set. If not, they are not!**
3. **Pick any valve inside each region. If the value satisfies the original inequality, then all of the numbers in the region are part of the solution set. If not, they are not!**

4. Write the solution set using interval notation and graph the solution set.

Question 01: Solve for x: $(x-3)(x+1) \geq 21$
Solution:

Step	Inequality	Reason
0	$(x-3)(x+1) \geq 21$	
1	$x^2 - 2x - 3 \geq 21$	
2	$x^2 - 2x - 24 \geq 0$	
3	$(x-6)(x+4) \geq 0$	
4	Determine Boundary Points: $(x+4)(x-6) = 0$ $x = -4 \;;\; x = 6$	
5	Check Boundary Points: 1. $x = -4 : ([-4]-3)([-4]+1) \overset{?}{\geq} 21$ $(-7)(-3) \overset{?}{\geq} 21$ $21 \geq 21$ True ; -4 is in the solution set 2. $x = 6 : ([6]-3)([6]+1) \overset{?}{\geq} 21$ $(3)(7) \overset{?}{\geq} 21$ $21 \geq 21$ True ; 6 is in the solution set	
6	Check Intervals: 1. $(-\infty, -4)$: Test Point $x : -6 : ([-6]-3)([-6]+1) \overset{?}{\geq} 21$ $(-9)(-5) \overset{?}{\geq} 21$ $45 \geq 21$ True ; $(-\infty, -4)$ is in the solution set 2. $(-4, 6)$: Test Point $x : 0 : ([0]-3)([0]+1) \overset{?}{\geq} 21$ $(-3)(1) \overset{?}{\geq} 21$ $-3 \geq 21$ False 3. $(6, +\infty)$: Test Point $x : 10 : ([10]-3)([10]+1) \overset{?}{\geq} 21$ $(7)(11) \overset{?}{\geq} 21$ $77 \geq 21$ True ; $(6, +\infty)$ is in the solution set	

| 7 | Solution Set: $(-\infty, -4] \cup [6, +\infty)$ | |

Graph of the solution set:

$$///////////] \overset{-4}{\rule{0pt}{0pt}} \underline{\qquad\qquad} \underset{0}{|} \underline{\qquad\qquad} \overset{6}{[} ///////////$$