Inequalities
Absolute Value

MATH by Wilson
Your Personal Mathematics Trainer
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There are two (2) types to be considered:

1. Type 1: $|\mathbf{u}| < \mathbf{b}$; $|\mathbf{u}| \le \mathbf{b}$ where $\mathbf{b} > 0$

The solution of $|\mathbf{u}| < \mathbf{b}$; $\mathbf{b} > 0$ **is** $-\mathbf{b} < \mathbf{u} < \mathbf{b}$ **- open interval**

The solution of $|\mathbf{u}| \le \mathbf{b}$; $\mathbf{b} > 0$ **is** $-\mathbf{b} \le \mathbf{u} \le \mathbf{b}$ **- closed interval**

Note: Solutions are open or closed intervals

Question 01: Solve for x: $|5\mathbf{x} - 3| < 6$

Solution:

Step	Equation	Reason
0	$\lvert 5\mathbf{x} - 3\rvert < 6$	
1	$-6 < 5\mathbf{x} - 3 < 6$	
2	$-3 < 5\mathbf{x} < 9$	
3	$-\frac{3}{5} < \mathbf{x} < \frac{9}{5}$	

In symbols, the solution is $\left(-\frac{3}{5}, \frac{9}{5}\right)$

Graph of the solution set:

2. Type 2: $|\mathbf{u}| > \mathbf{b}$; $|\mathbf{u}| \geq \mathbf{b}$ where $\mathbf{b} > 0$

The solution of $|\mathbf{u}| > \mathbf{b}$; $\mathbf{b} > 0$ **satisfies** $\mathbf{u} < -\mathbf{b}$ or $\mathbf{u} > \mathbf{b}$ **:**

$$(-\infty, -\mathbf{b}) \cup (\mathbf{b}, +\infty)$$

The solution of $|\mathbf{u}| \geq \mathbf{b}$; $\mathbf{b} > 0$ **satisfies** $\mathbf{u} \leq -\mathbf{b}$ or $\mathbf{u} \geq \mathbf{b}$

$$(-\infty, -\mathbf{b}] \cup [\mathbf{b}, +\infty)$$

Note: Solutions are unions of open or closed intervals

Question 02: Solve for x: $|2 + 3\mathbf{x}| \geq 7$

Solution:

Step	Equation	Reason
0	$\lvert 2 + 3\mathbf{x} \rvert \geq 7$	
1	$2 + 3\mathbf{x} \leq -7$ $3\mathbf{x} \leq -9$ $\mathbf{x} \leq -3$ $\Vert$ $2 + 3\mathbf{x} \geq 7$ $3\mathbf{x} \geq 5$ $\mathbf{x} \geq \frac{5}{3}$	

In symbols, the solution is $(-\infty, -3] \cup \left[\frac{5}{3}, +\infty\right)$

Graph of the solution set: