FUNctions: Intercept Points

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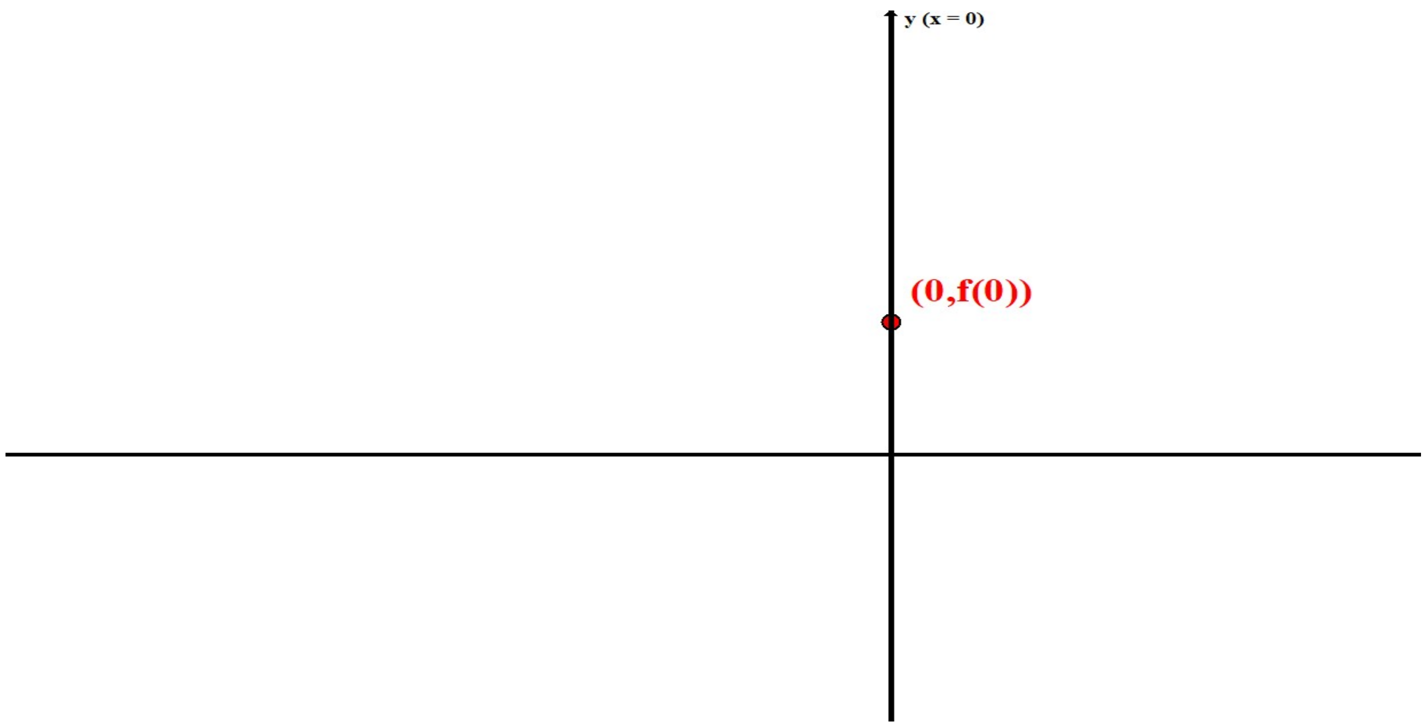
Definition: If $0 \in \mathbf{Dom\ f}$, then *the* point $(0, \mathbf{f}(0))$ is the y-intercept point of the function:



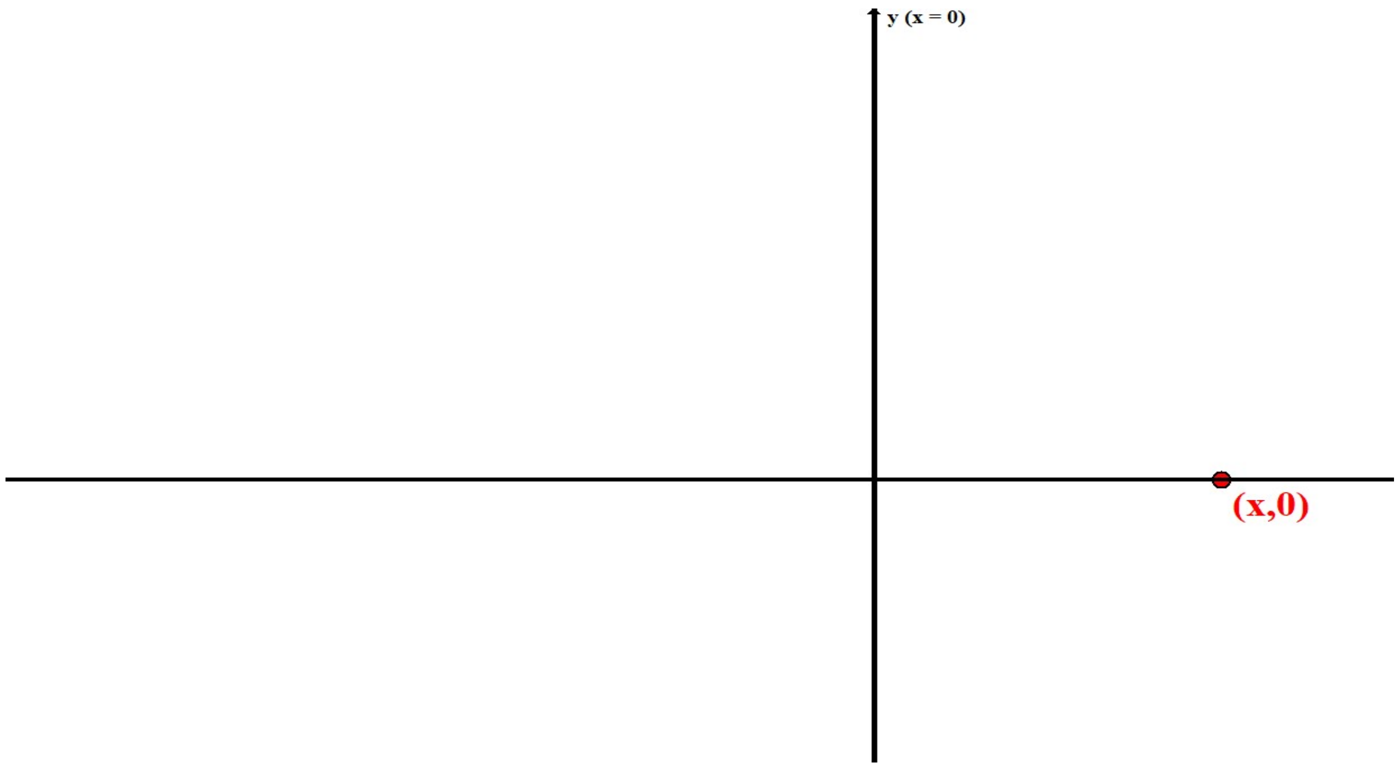
Note:

1. There is a maximum of one (1) y-intercept point

2. It is the intersection of the graph with the y-axis

3. The Action Verb is EVALUATE – Calculate $\mathbf{f}(0) : (0, \mathbf{f}(0))$

Definition: If $\mathbf{f}(\mathbf{x}) = 0$ for some x in the domain of a function $\mathbf{f}$, then the point $(\mathbf{x}, 0)$ is *an* x-intercept point:



Note:

1. There are 0,1,2,…n,… up to an infinite number of x-intercept points:
   $\left\{\mathbf{x} \in \mathbf{Dom\ f} \,\middle|\, \mathbf{f}(\mathbf{x}) = 0\right\}$

2. It is an intersection of the graph with the x-axis

3. The Action Verb is Solve: Solve the equation $\mathbf{f}(\mathbf{x}) \overset{\mathbf{SET}}{=} 0$

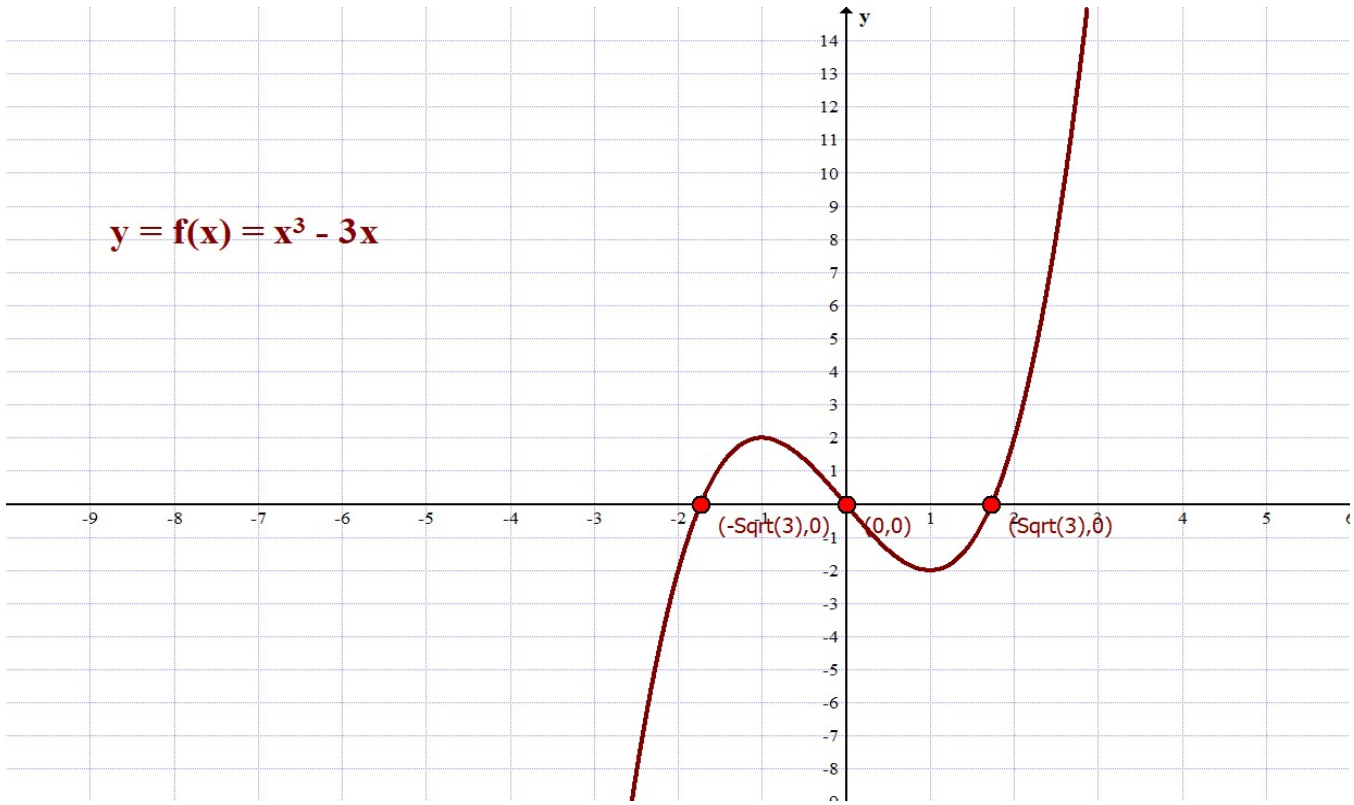
In the example below, we draw the entire graph but at this point of the analysis we only know the intercept points.

Example 01: Find the x-intercept and y-intercept points of the following functions:

1. $\mathbf{f(x) = x^3 - 3x}$

   The y-intercept point is $(0,0)$. The x-intercept points are given by

   $$\mathbf{f(x) = x}\left(\mathbf{x}^2 - 3\right) \overset{\textbf{SET}}{=} 0 \Rightarrow \mathbf{x} = \pm\sqrt{3}, 0 \Rightarrow \left(-\sqrt{3}, 0\right), (0,0), \left(\sqrt{3}, 0\right)$$

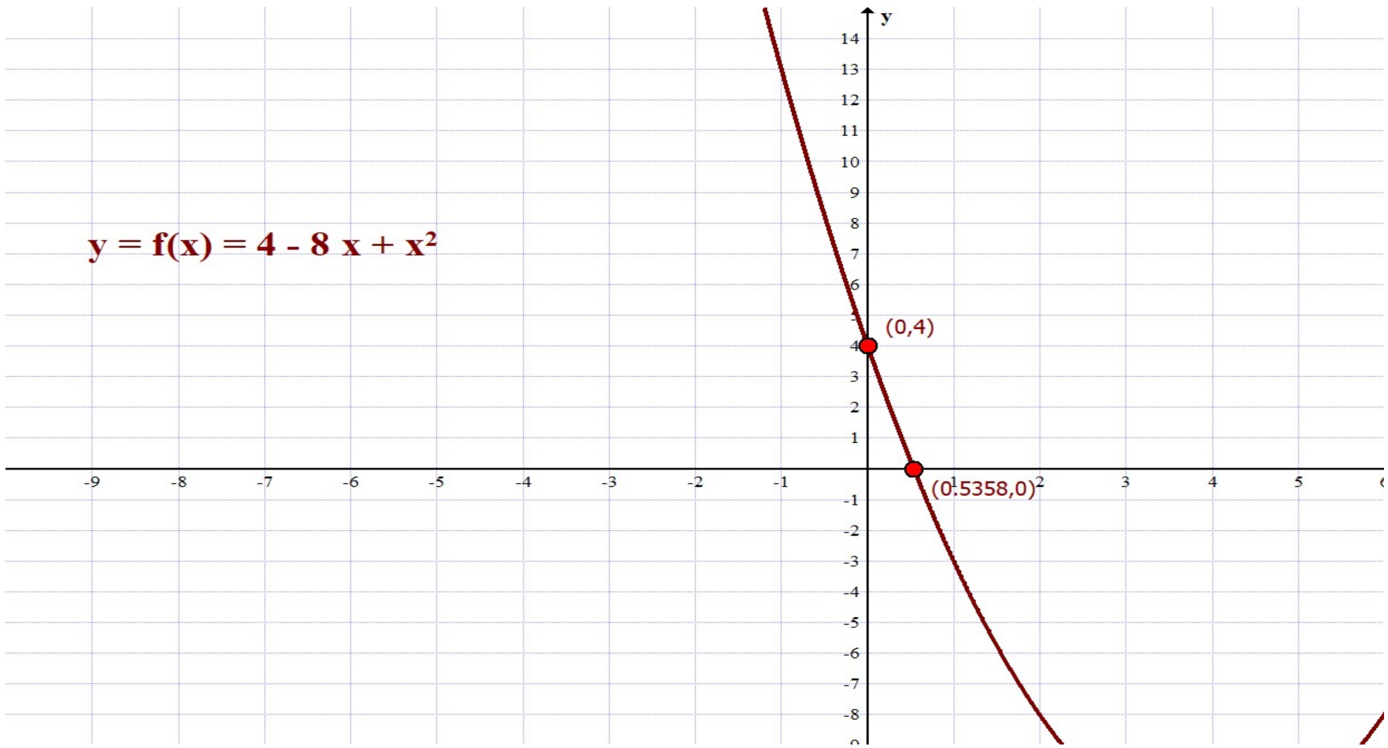


2. $\mathbf{f}(\mathbf{x}) = 4 - 8\mathbf{x} + \mathbf{x}^2$

The y-intercept point is $(0,4)$. The x-intercept points are given by

$$\mathbf{f}(\mathbf{x}) = 4 - 8\mathbf{x} + \mathbf{x}^2 \overset{\text{SET}}{=} 0 \Rightarrow \mathbf{x} = \frac{8 \pm \sqrt{(-8)^2 - 4(1)(4)}}{2} = \frac{8 \pm \sqrt{48}}{2} = \frac{8 \pm 4\sqrt{3}}{2}$$

$$\Rightarrow \mathbf{x} = 4 \pm 2\sqrt{3} \Rightarrow \left(4 - 2\sqrt{3}, 0\right), \left(4 + 2\sqrt{3}, 0\right)$$

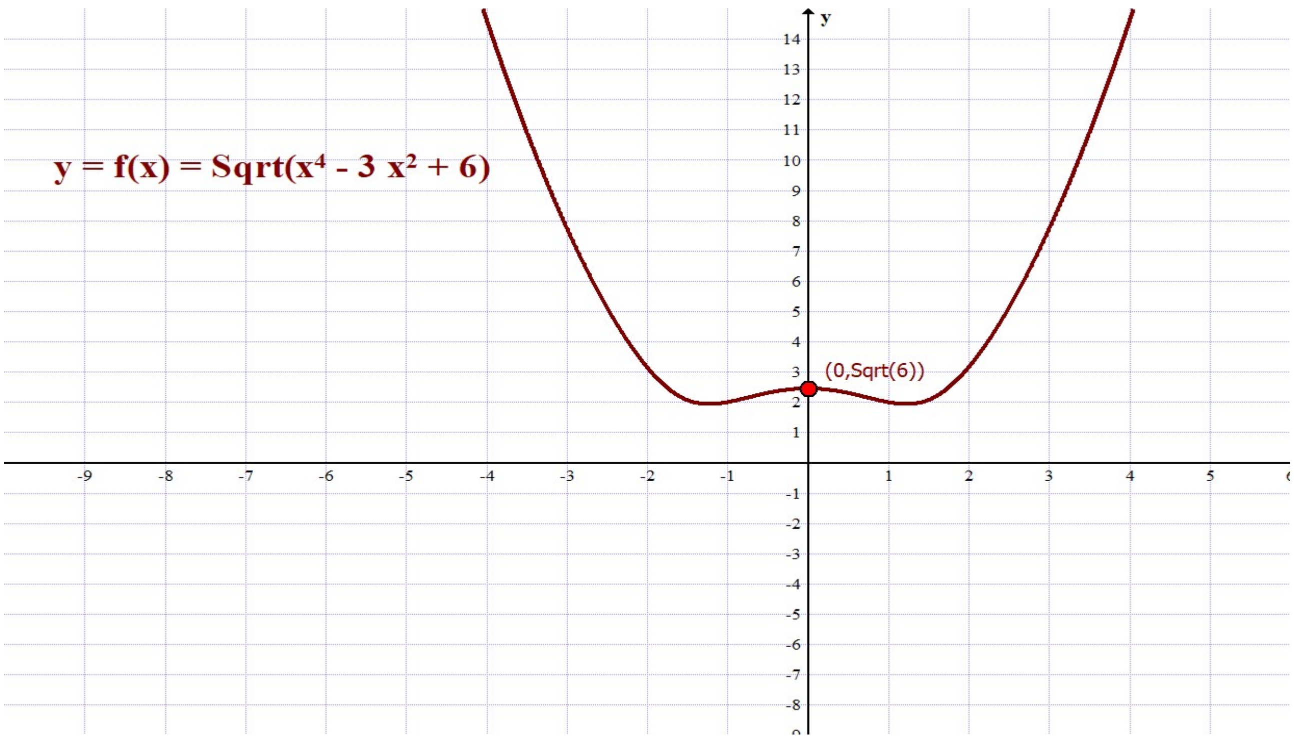


3. $\mathbf{f}(\mathbf{x})=\sqrt{\mathbf{x}^4-3\mathbf{x}^2+6}$

The y-intercept point is $(0,\sqrt{6})$. The x-intercept points are given by

$$\mathbf{f}(\mathbf{x})=\sqrt{\mathbf{x}^4-3\mathbf{x}^2+6}\overset{\text{SET}}{=}0\Rightarrow\mathbf{x}^4-3\mathbf{x}^2+6=0\Rightarrow\mathbf{x}^2=\frac{3\pm\sqrt{(-3)^2-4(1)(6)}}{2}$$

$$\Rightarrow\mathbf{x}^2=\frac{3\pm\sqrt{-15}}{2}\in\mathbb{C}\Rightarrow\text{No x-intercept points}$$



4. $\mathbf{f}(\mathbf{x}) = \dfrac{2\mathbf{x}}{\mathbf{x}^2 + 4}$

The y-intercept point is $(0,0)$. The x-intercept points are given by

$$\mathbf{f}(\mathbf{x}) = \frac{2\mathbf{x}}{\mathbf{x}^2 + 4} \overset{\text{SET}}{=} 0 \Rightarrow \mathbf{x} = 0 \Rightarrow (0,0)$$ Both an x-intercept and y-intercept point