FUNctions
Odd/Even

MATH by Wilson
Your Personal Mathematics Trainer
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Definition: A function $\mathbf{f}$ is **even** if $\mathbf{f(-x) = f(x)}$ for all $\mathbf{x \in Dom\ f}$

Note: An *even* function will have a graph that is symmetric with respect to the y-axis.

Definition: A function $\mathbf{f}$ is **odd** if $\mathbf{f(-x) = -f(x)}$ for all $\mathbf{x \in Dom\ f}$

Note: An *odd* function will have a graph that is symmetric with respect to the Origin: $(0,0)$

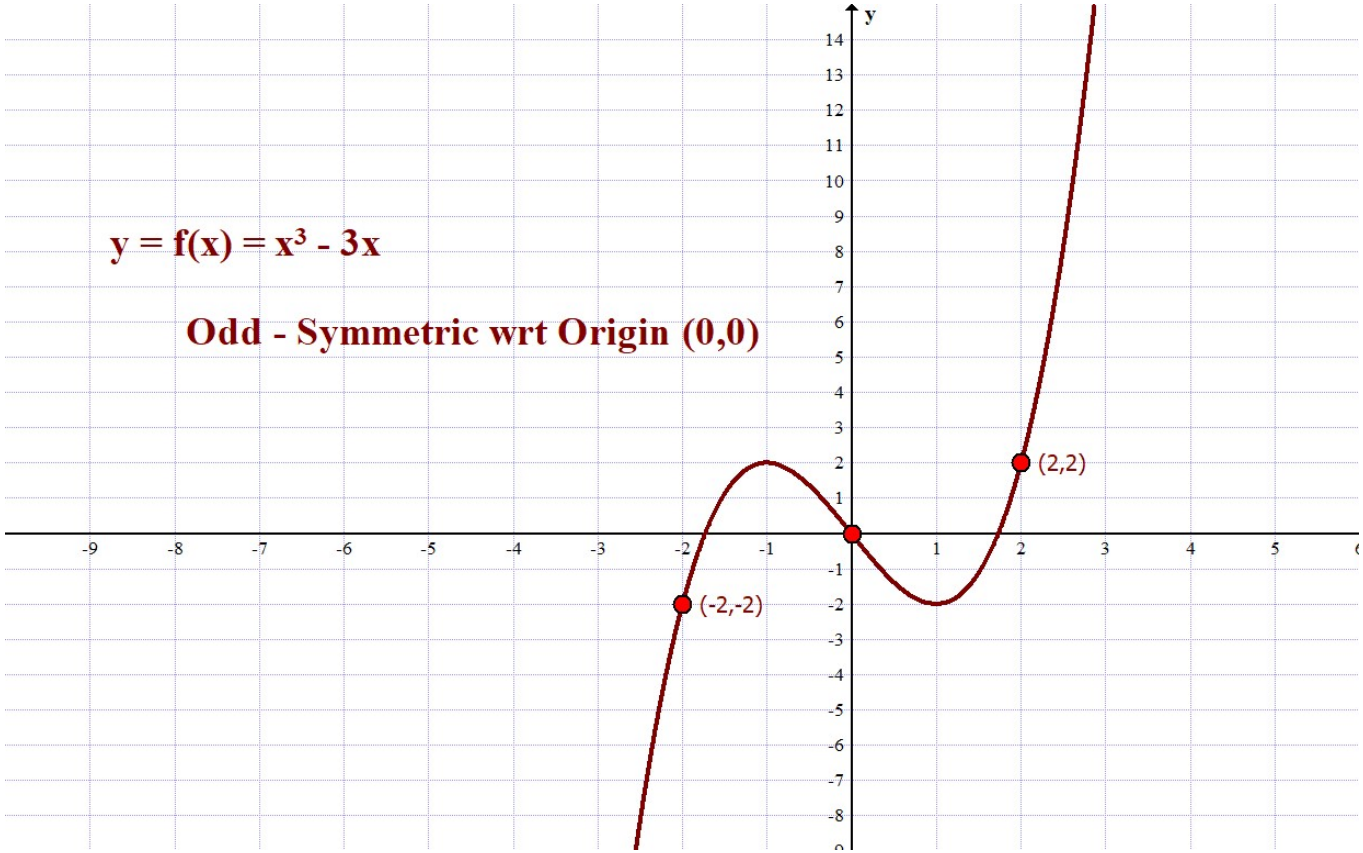
We will test the following functions to determine if they are even, odd, or neither and then present their graphs to visually verify the appropriate symmetry.

Example 01: Check the following functions to see if they are even, odd, or neither.

a. $\mathbf{f(x) = x^3 - 3x}$

We have $\mathbf{f(-x) = (-x)^3 - 3(-x) = -x^3 + 3x = -(x^3 - 3x) = -f(x) \Rightarrow ODD}$

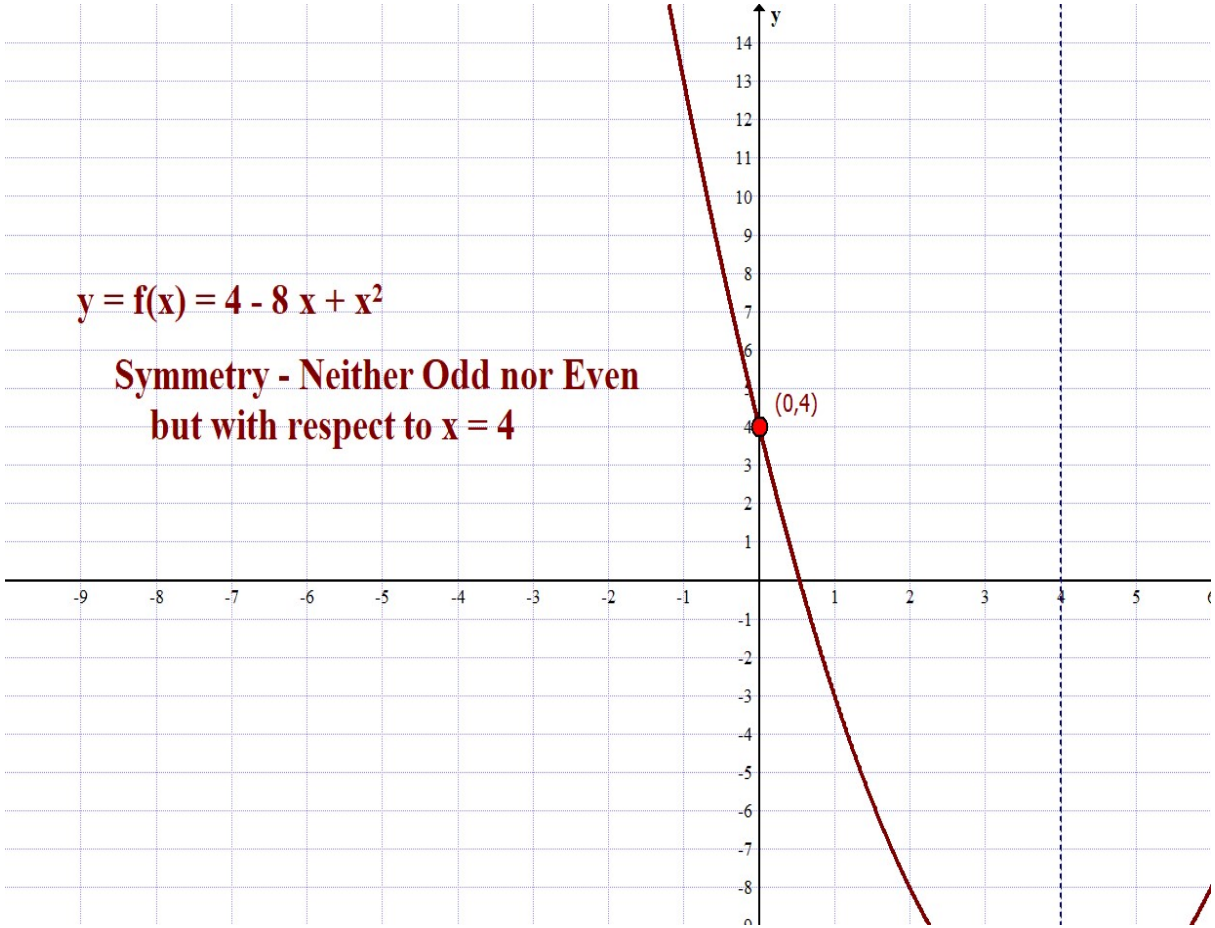
We know the graph is symmetric with respect to the Origin (0,0) as its graph below shows:



b. $\mathbf{f(x) = 4 - 8x + x^2}$

We have $\mathbf{f(-x) = 4 - 8(-x) + (-x)^2 = 4 + 8x + x^2 \neq \begin{cases} f(x) \\ -f(x) \end{cases} \Rightarrow Neither}$

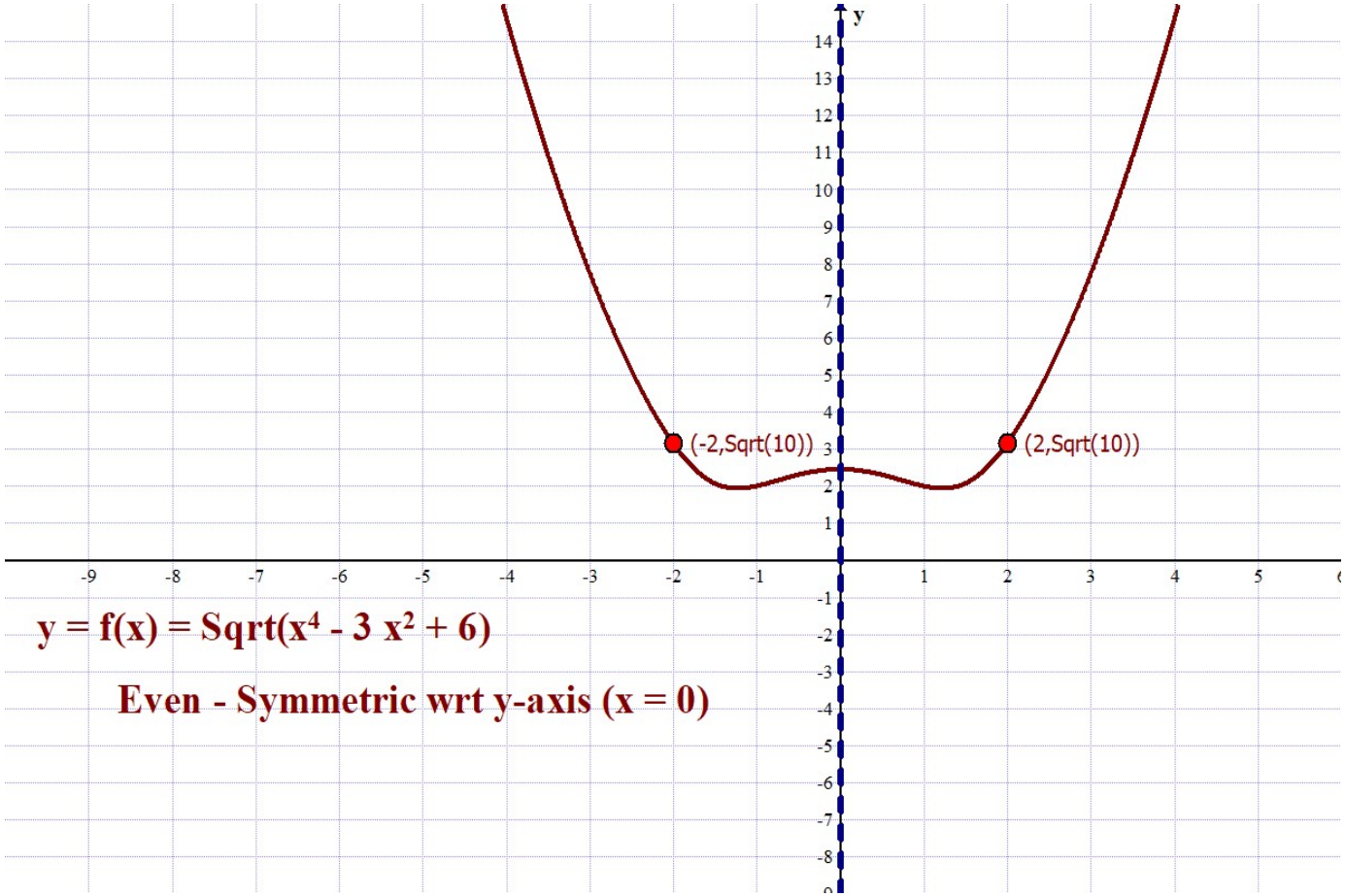
We know the graph is NOT symmetric with respect to the Origin (0,0) or the y-axis (x = 0) as shown below:



c. $\mathbf{f(x) = \sqrt{x^4 - 3x^2 + 6}}$

We have $\mathbf{f(-x) = \sqrt{(-x)^4 - 3(-x)^2 + 6} = \sqrt{x^4 - 3x^2 + 6} = f(x) \Rightarrow}$ **EVEN**

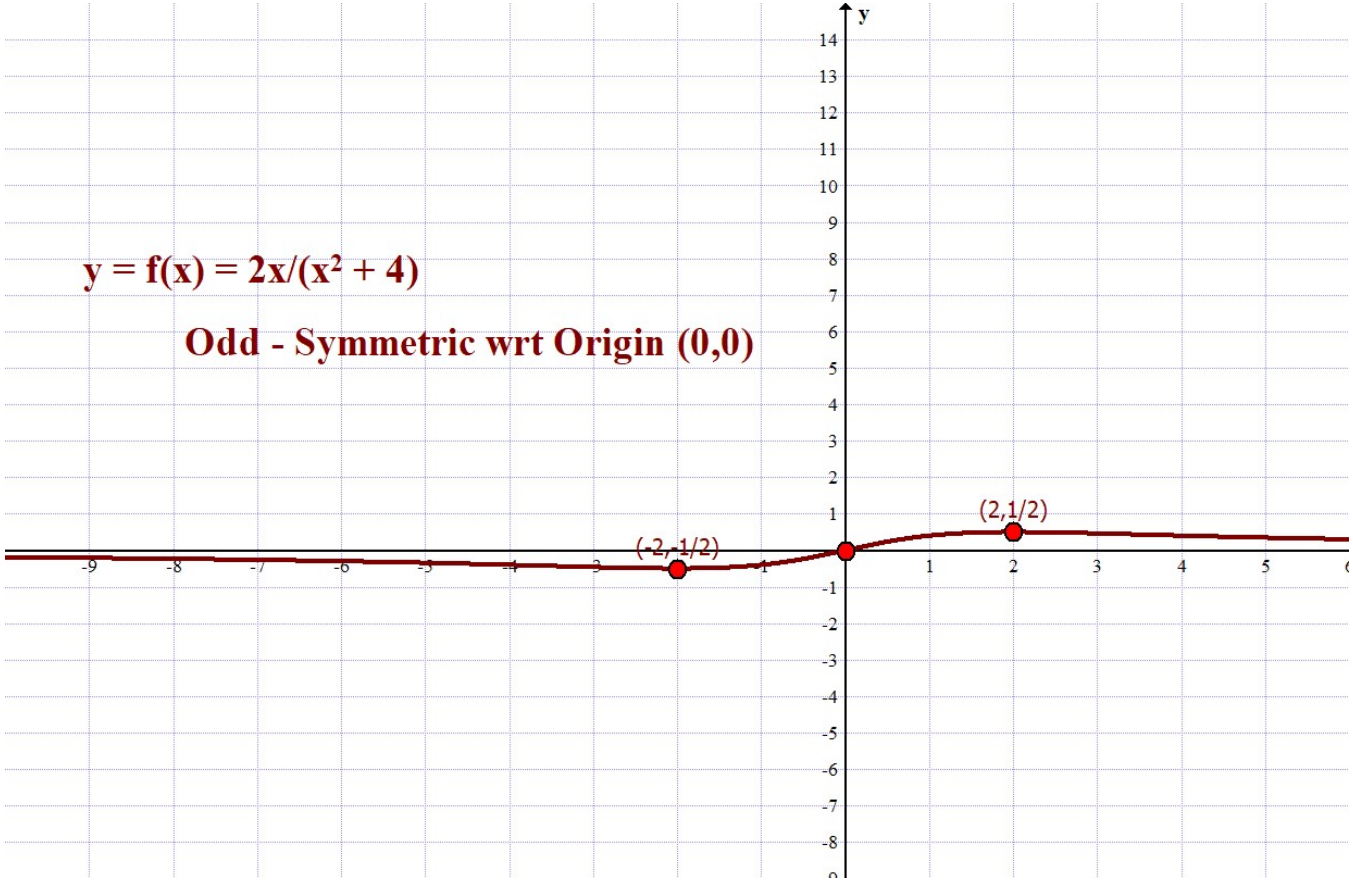
We know the graph is symmetric with respect to the y-axis (x = 0) as shown below:



d. $\mathbf{f(x)=\dfrac{2x}{x^2+4}}$

We have $\mathbf{f(-x)=\dfrac{2(-x)}{(-x)^2+4}=\dfrac{-2x}{x^2+4}=-\dfrac{2x}{x^2+4}=-f(x)\Rightarrow ODD}$

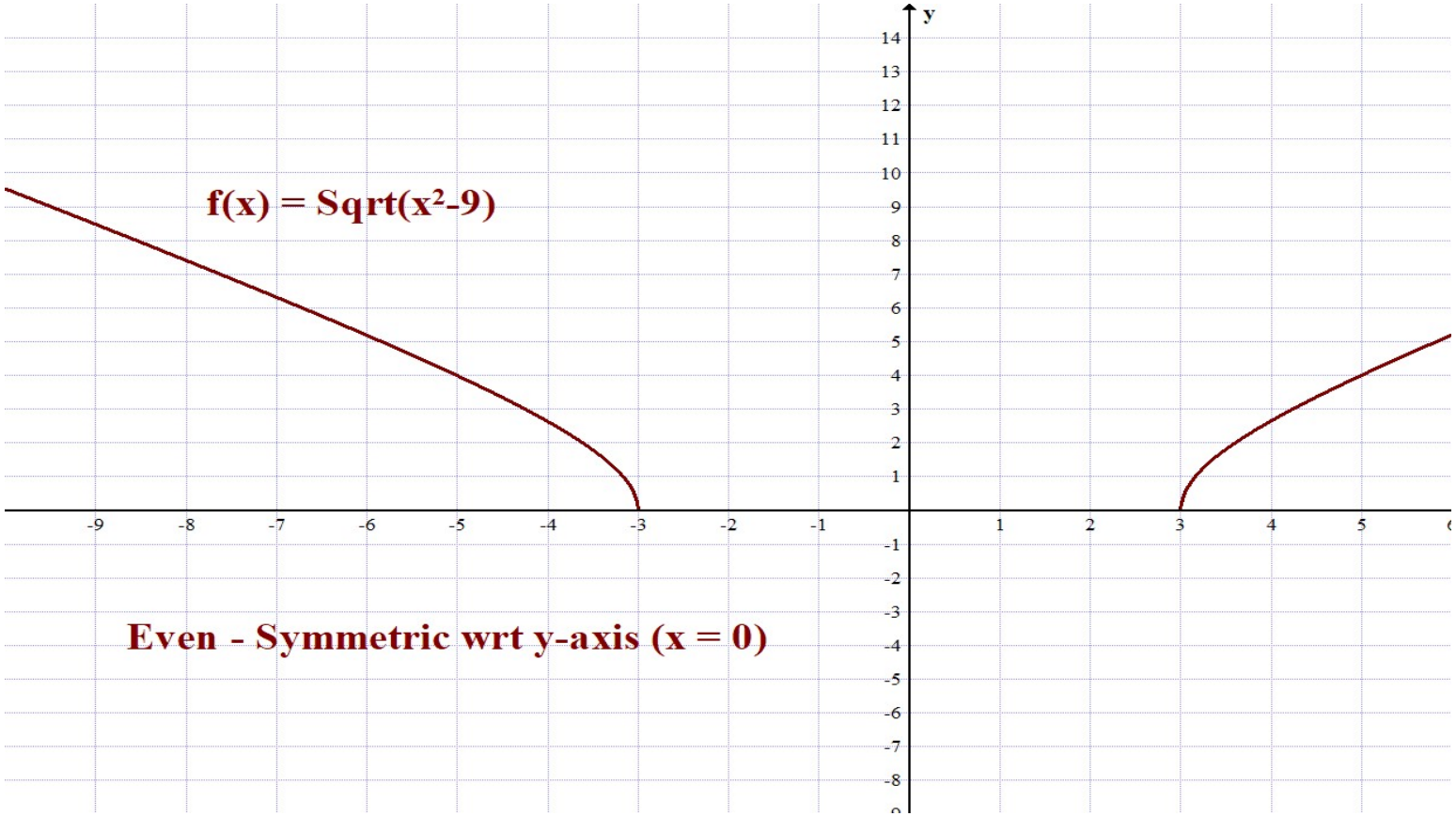
We know the graph is symmetric with respect to the Origin (0,0) as shown below:



e. $\mathbf{f(x) = \sqrt{x^2 - 9}}$

We have $\mathbf{f(-x) = \sqrt{(-x)^2 - 9} = \sqrt{x^2 - 9} = f(x) \Rightarrow}$ **EVEN**

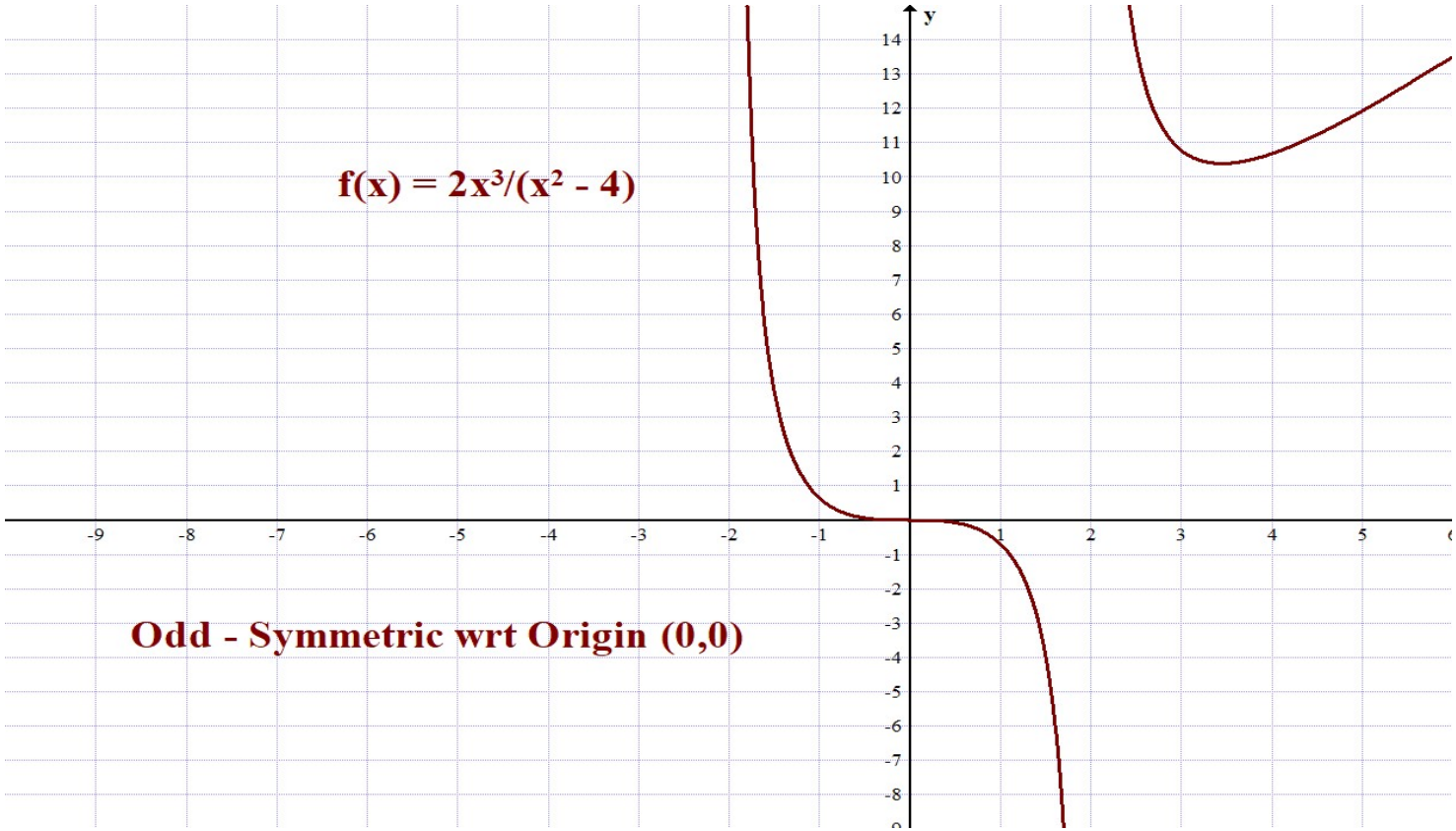
We know the graph is symmetric with respect to the y-axis as shown below:



f. $\mathbf{f(x)=\dfrac{2x^3}{x^2-4}}$

We have $\mathbf{f(-x)=\dfrac{2(-x)^3}{(-x)^2-4}=-\dfrac{2x^3}{x^2-4}=-f(x)\Rightarrow ODD}$

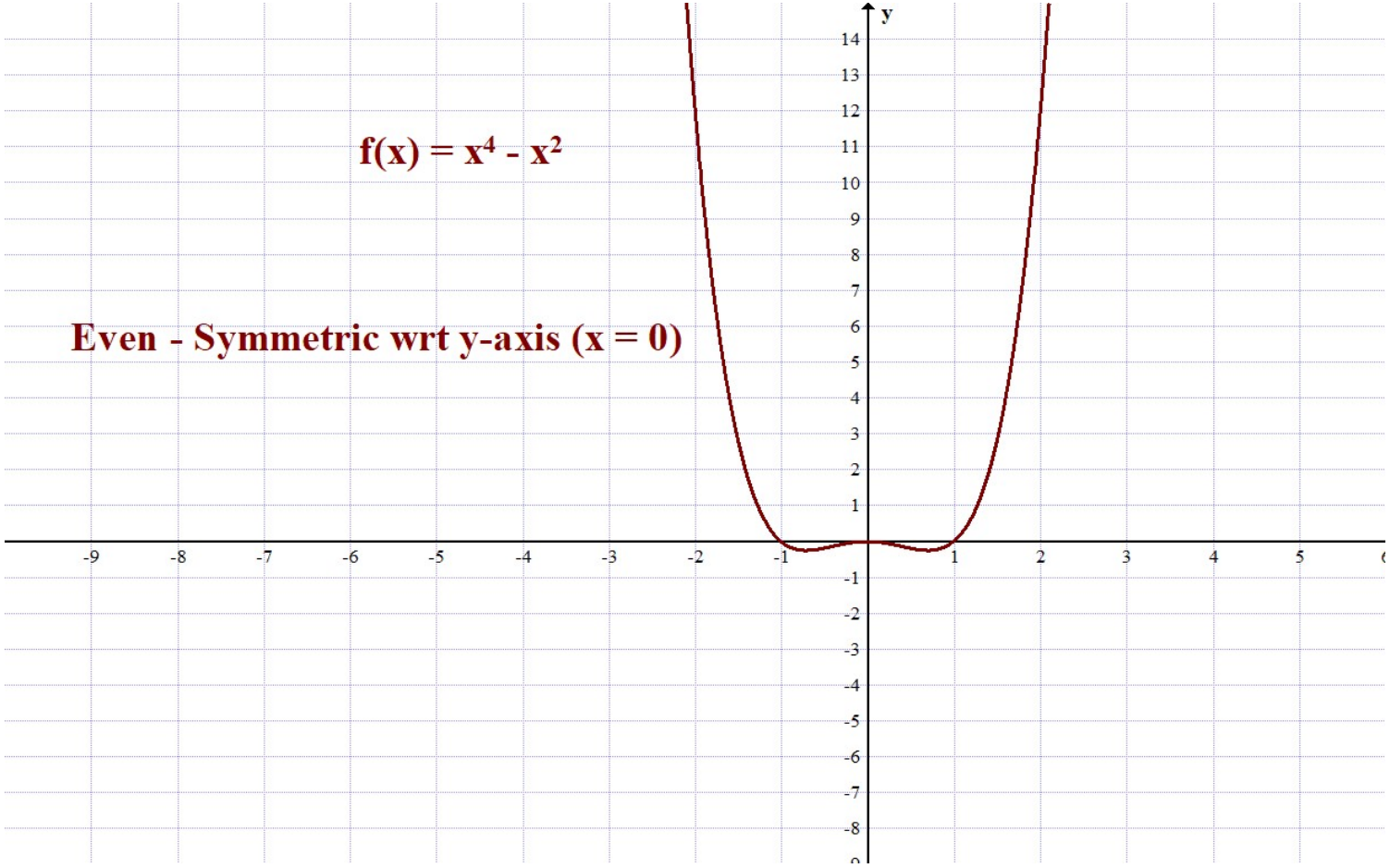
We know the graph is symmetric with respect to Origin (0,0) as shown below:



g. $\mathbf{f(x) = x^4 - 2x^2}$

We have $\mathbf{f(-x) = (-x)^4 - (-x)^2 = x^4 - x^2 = f(x) \Rightarrow EVEN}$

We know the graph is symmetric with respect to the y-axis as shown below:



h. $\mathbf{f(x) = 3 - 5x^3 + 3x^5}$

We have $\mathbf{f(-x) = 3 - 5(-x)^3 + 3(-x)^5 = 3 + 5x^3 - 3x^5 \neq \begin{cases} f(x) \\ -f(x) \end{cases} \Rightarrow}$ **Neither**

We know the graph is NOT even or odd as shown below: