FUNctions: Increasing/Decreasing

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Your Personal Mathematics Trainer
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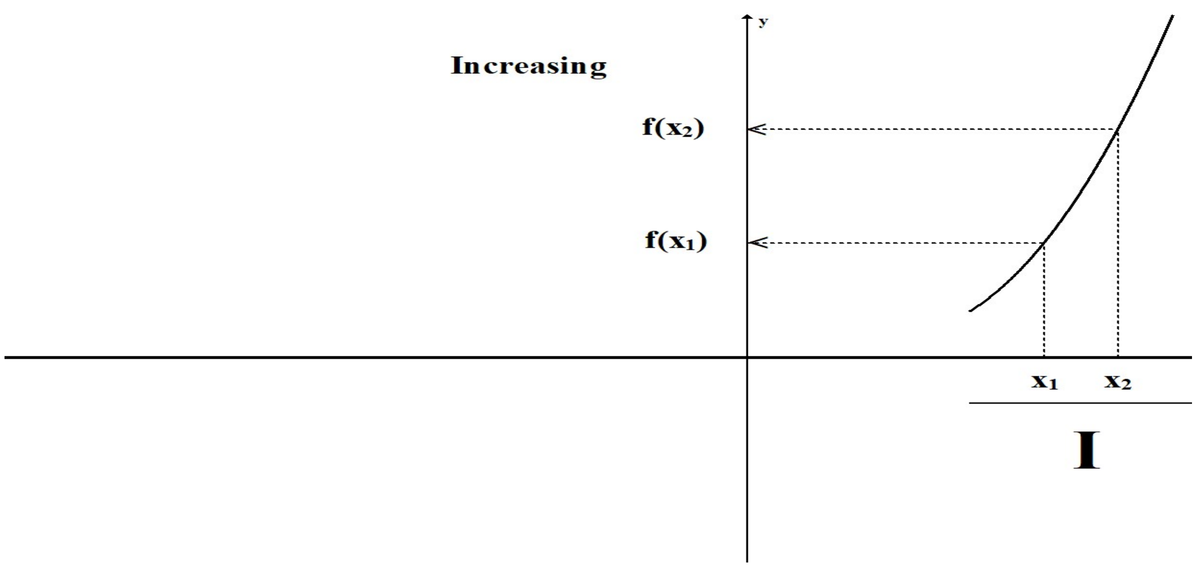
We usually analyze a function f "left to right". If the f(x) values are getting bigger (smaller) on an interval I, we say that the function is increasing (decreasing) on this set.

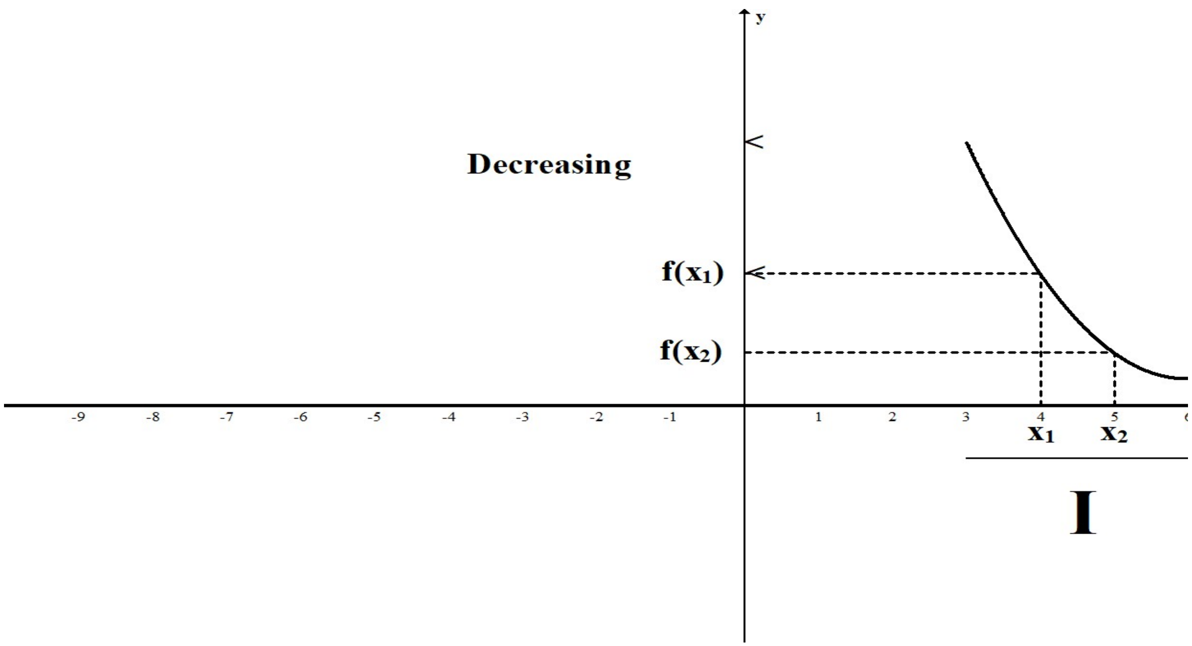
Definition: A function $\mathbf{f}$ is **increasing** (strictly) on an interval I if $\mathbf{f}(\mathbf{x}_2) > \mathbf{f}(\mathbf{x}_1)$ for $\mathbf{x}_1, \mathbf{x}_2 \in \mathbf{I}$ and $\mathbf{x}_1 < \mathbf{x}_2$

Note: If $\mathbf{f}(\mathbf{x}_2) \geq \mathbf{f}(\mathbf{x}_1)$, then f is non-decreasing.



Definition: A function $\mathbf{f}$ is **decreasing** (strictly) on an interval I if $\mathbf{f}(\mathbf{x}_2) < \mathbf{f}(\mathbf{x}_1)$ for $\mathbf{x}_1, \mathbf{x}_2 \in \mathbf{I}$ and $\mathbf{x}_1 < \mathbf{x}_2$

Note: If $\mathbf{f}(\mathbf{x}_2) \leq \mathbf{f}(\mathbf{x}_1)$, then f is non-increasing.



If we have the graph of f , we can easily determine the increasing/decreasing intervals. The following examples will list the increasing/decreasing intervals for the given graphs. We will denote the increasing intervals by Inc f and the decreasing intervals by Dec f.

However, when only the formula for the function f is given, we usually need advanced techniques to determine the increasing/decreasing intervals and that will NOT be a subject considered in College Algebra.

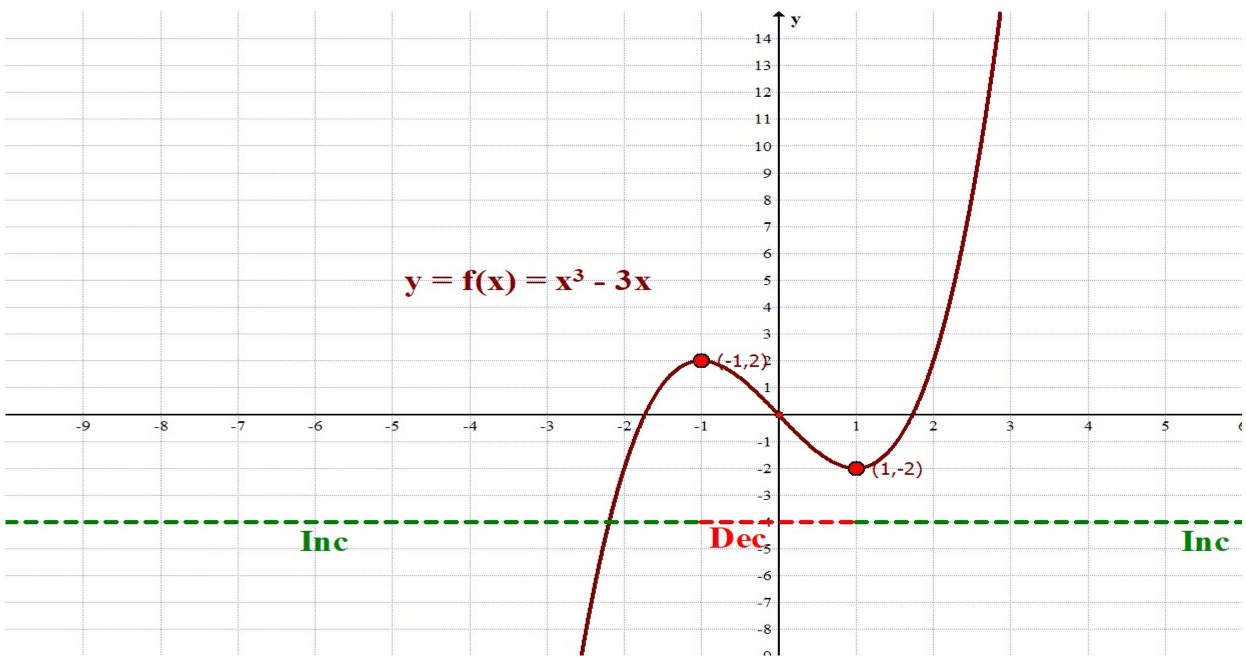
Example 01: Determine where the following functions are increasing (**Inc f**) and decreasing (**Dec f**) using the given functions and their graphs:

1. $\mathbf{f(x) = x^3 - 3x}$

   Considering the graph, we have

   a. $\mathbf{Inc\ f} = (-\infty, -1] \cup [1, +\infty)$

   b. $\mathbf{Dec\ f} = [-1, 1]$

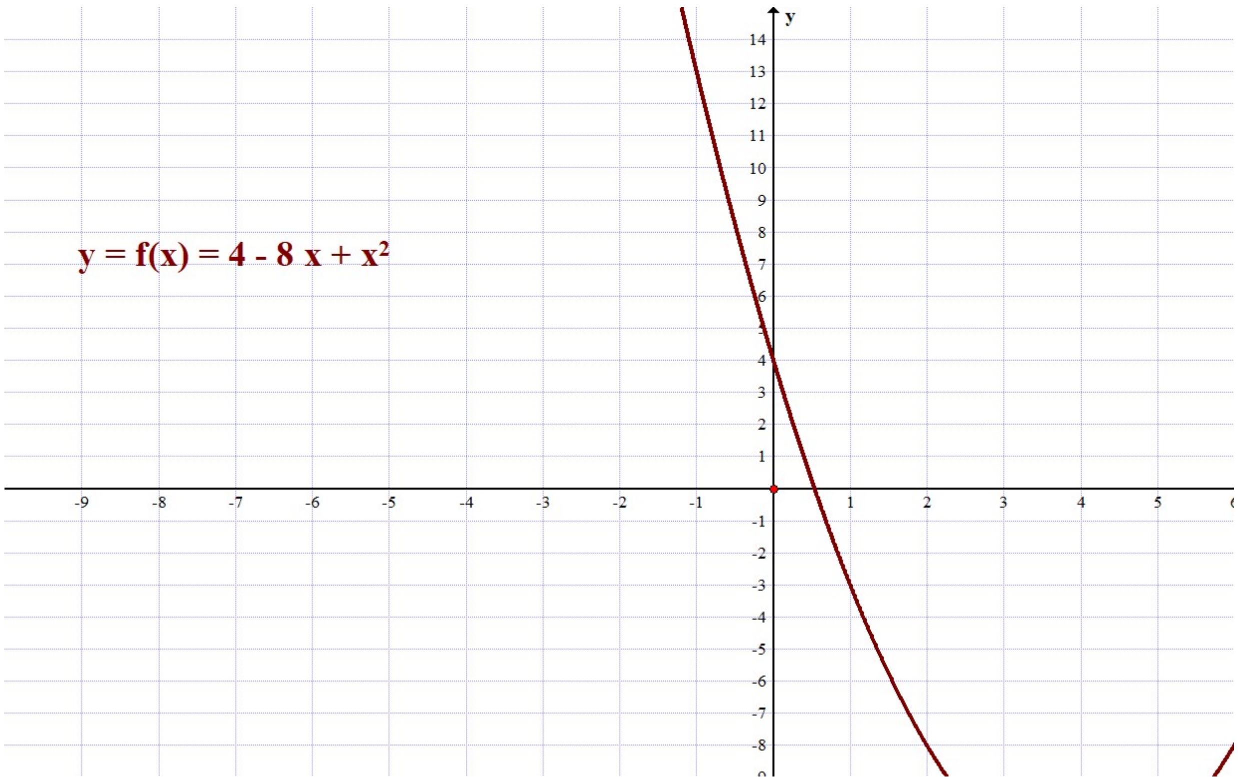


2. $\mathbf{f}(\mathbf{x}) = 4 - 8\mathbf{x} + \mathbf{x}^2$

From the graph, we obtain

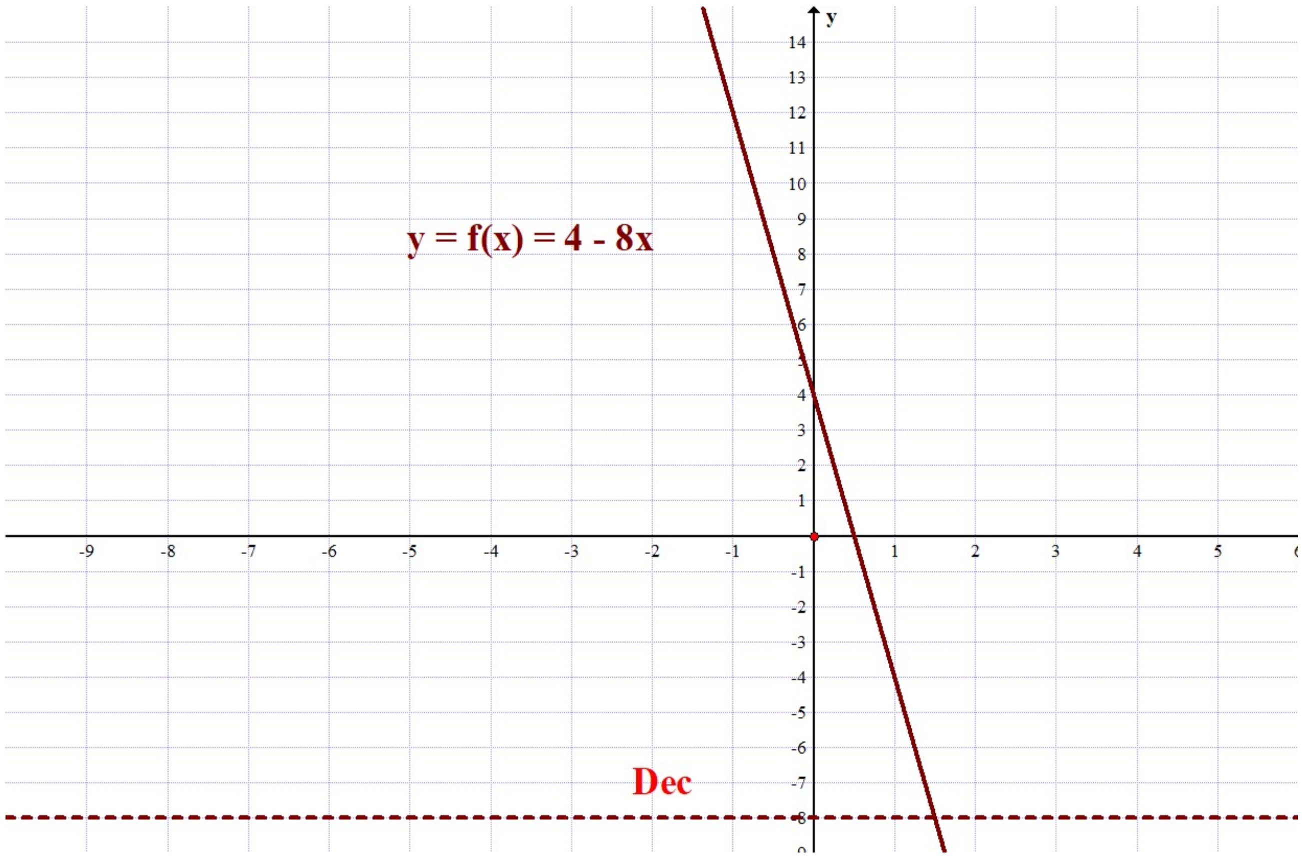
a. **Inc f** $= [4, +\infty)$

b. **Dec f** $= (-\infty, 4]$



3. $\mathbf{f}(\mathbf{x}) = 4 - 8\mathbf{x}$

We have $\textbf{Dec f} = (-\infty, +\infty)$

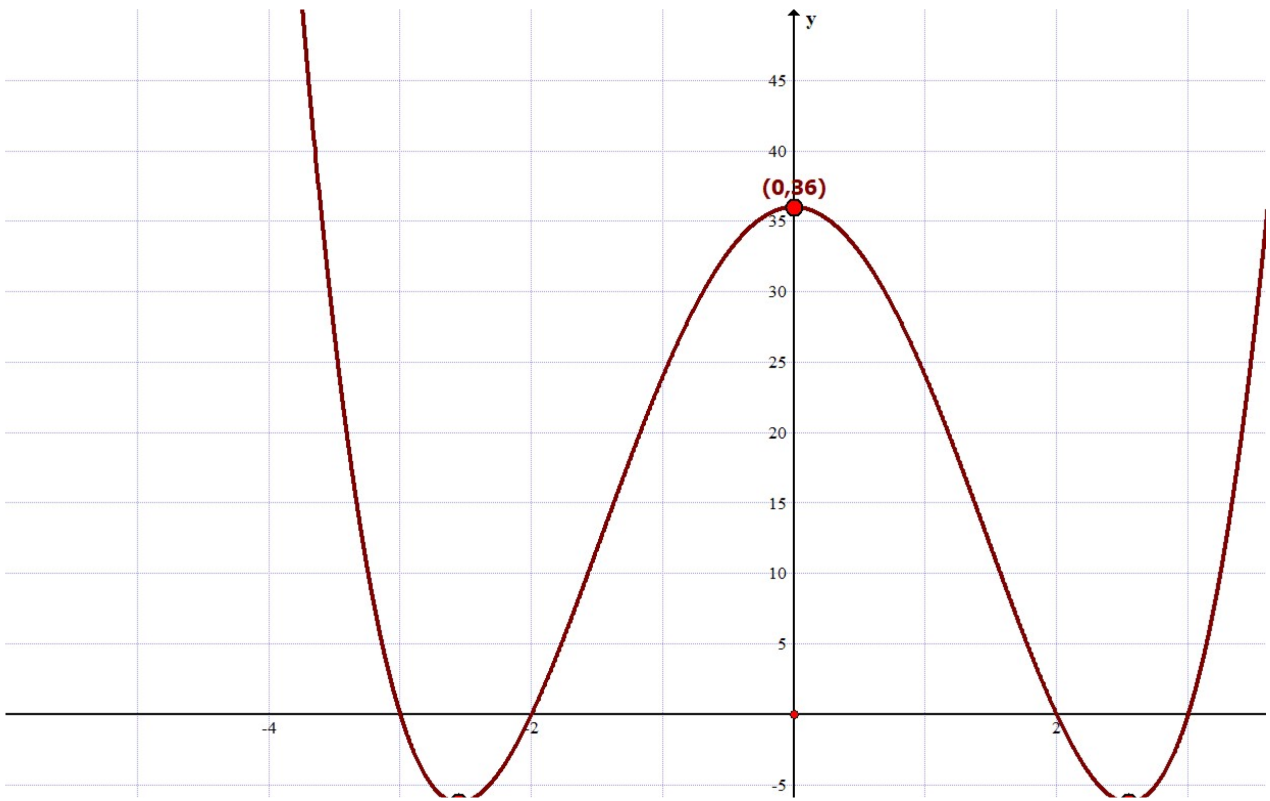


4. $\mathbf{f}(\mathbf{x}) = \mathbf{x}^4 - 13\mathbf{x}^2 + 36$

The graph yields

a. **Inc f** $= \left[-\sqrt{13/2}, 0\right] \cup \left[\sqrt{13/2}, +\infty\right)$

b. **Dec f** $= \left(-\infty, -\sqrt{13/2}\right] \cup \left[0, \sqrt{13/2}\right]$



5. $\mathbf{f}(\mathbf{x}) = \dfrac{2\mathbf{x}^3}{\mathbf{x}^2 - 4}$

Looking at the graph, we have

a. **Inc f** $= \left(-\infty, -2\sqrt{3}\right] \cup \left[2\sqrt{3}, +\infty\right)$

b. **Dec f** $= \left[-2\sqrt{3}, -2\right) \cup \left(-2, 2\right) \cup \left(2, 2\sqrt{3}\right]$