FUNctions: Maximum/Minimum Points

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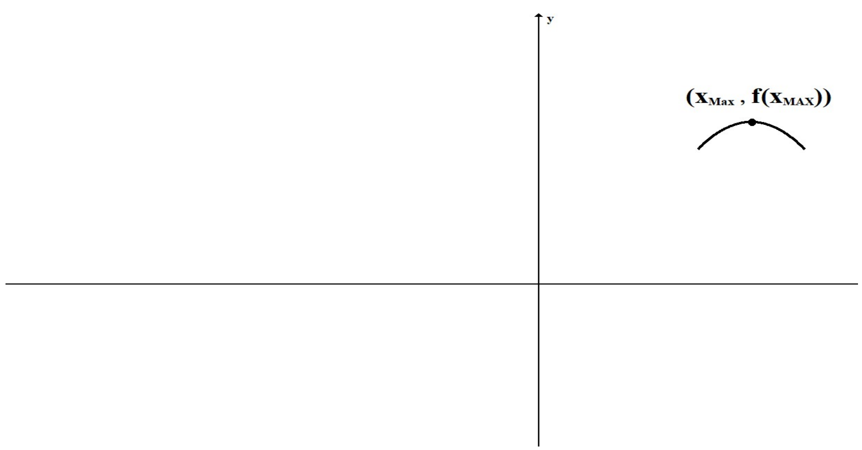
When a function f is increasing (decreasing) until it reaches a point and then decreases (increases) we have a maximum (minimum) point on the graph.

Definition: A point $(\mathbf{x_{MAX}}, \mathbf{f}(\mathbf{x_{MAX}}))$ on the graph of a function $\mathbf{f}$ is called a **relative maximum point** of f if

$$\mathbf{f}(\mathbf{x_{MAX}}) \geq \mathbf{f}(\mathbf{x})$$

for x "close to" $\mathbf{x_{MAX}}$

Note: If $\mathbf{f}(\mathbf{x_{MAX}}) \geq \mathbf{f}(\mathbf{x})$ for all $\mathbf{x} \in \mathbf{Dom\ f}$, the point $(\mathbf{x_{MAX}}, \mathbf{f}(\mathbf{x_{MAX}}))$ is called an **absolute minimum point** of f.

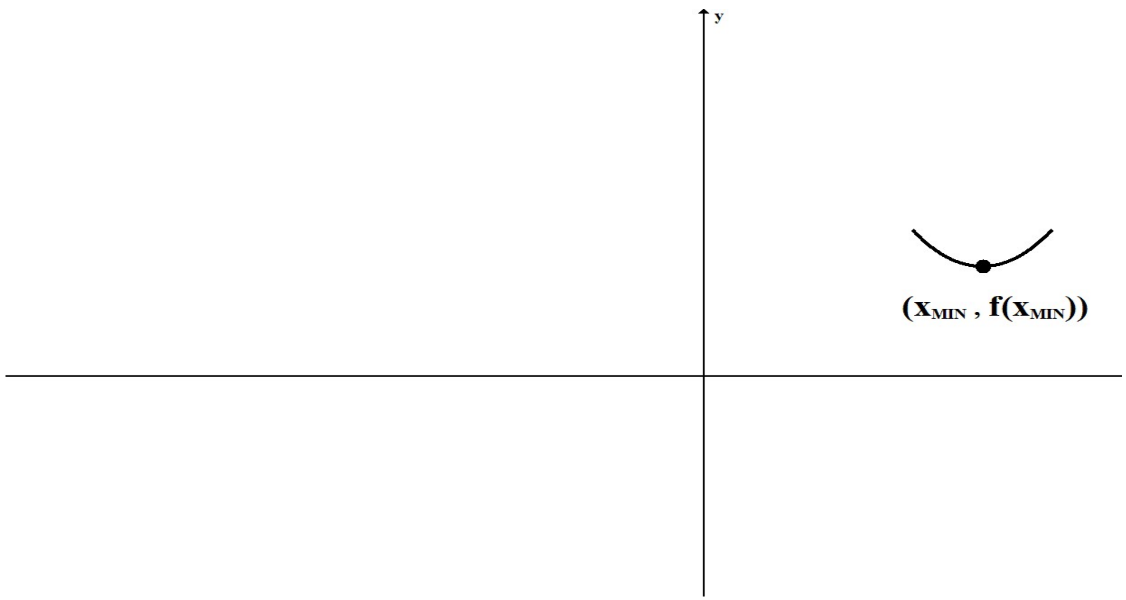


Definition: A point $(\mathbf{x_{MIN}}, \mathbf{f}(\mathbf{x_{MIN}}))$ on the graph of a function $\mathbf{f}$ is called a **relative maximum point** of f if

$$\mathbf{f}(\mathbf{x_{MIN}}) \le \mathbf{f}(\mathbf{x})$$

for x "close to" $\mathbf{x_{MIN}}$

Note: If $\mathbf{f}(\mathbf{x_{MIN}}) \le \mathbf{f}(\mathbf{x})$ for all $\mathbf{x} \in \mathbf{Dom\ f}$, the point $(\mathbf{x_{MIN}}, \mathbf{f}(\mathbf{x_{MIN}}))$ is called an **absolute minimum point** of f.



Note: A function can have 0, 1, 2, … , n, … up to an infinite number of relative (absolute) maximum (minimum) points.

Note: There can be more than one (1) absolute maximum (minimum) point but all of them *mus*t have the same y-value.

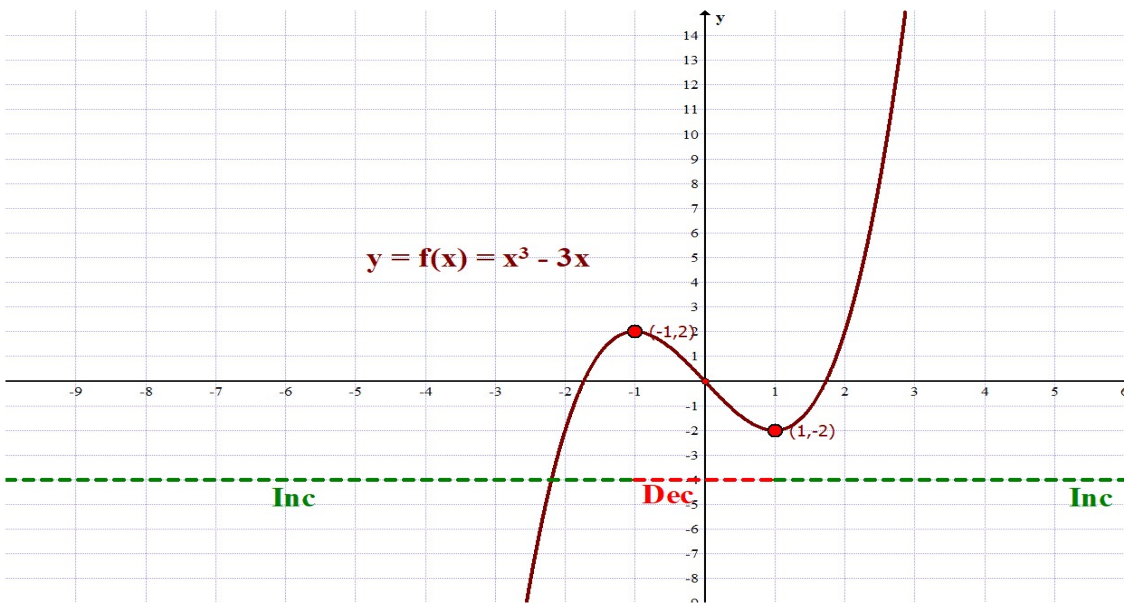
Example 01: From the graph, determine where the following functions have maximum and minimum points.

1. $\mathbf{f(x) = x^3 - 3x}$

   Considering the graph, we have

   a. Relative Maximum Point: $(-1, 2)$

   b. Relative Minimum Point: $(1, -2)$

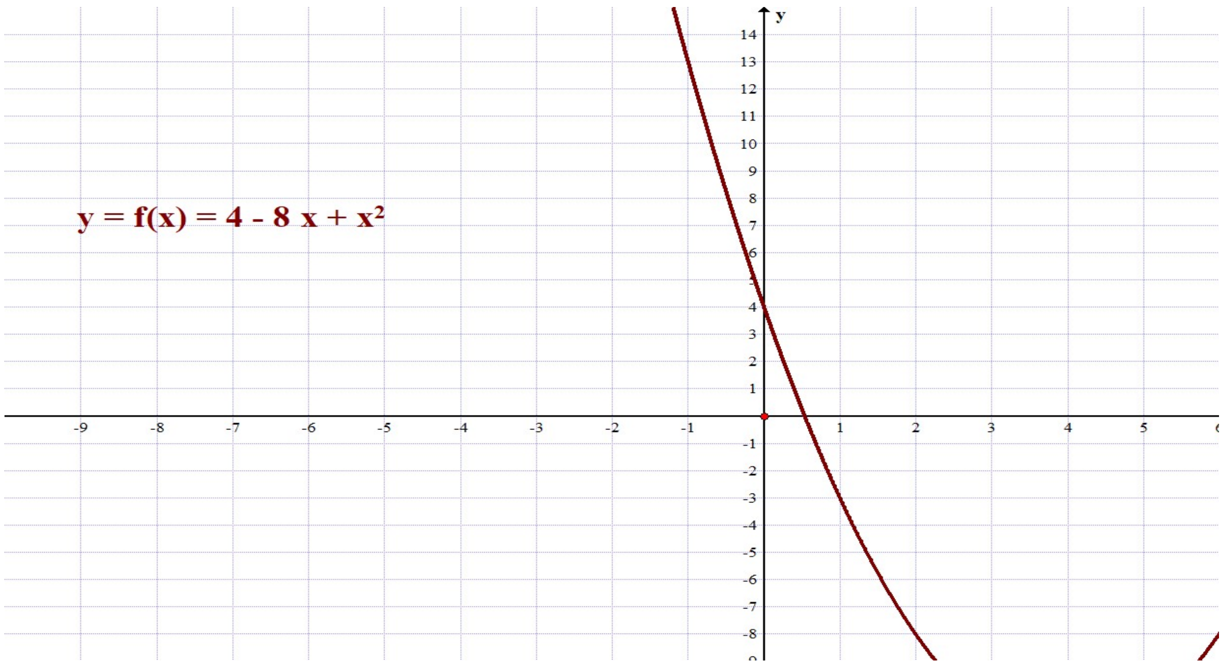


2. $\mathbf{f}(\mathbf{x}) = 4 - 8\mathbf{x} + \mathbf{x}^2$

From the graph, we obtain

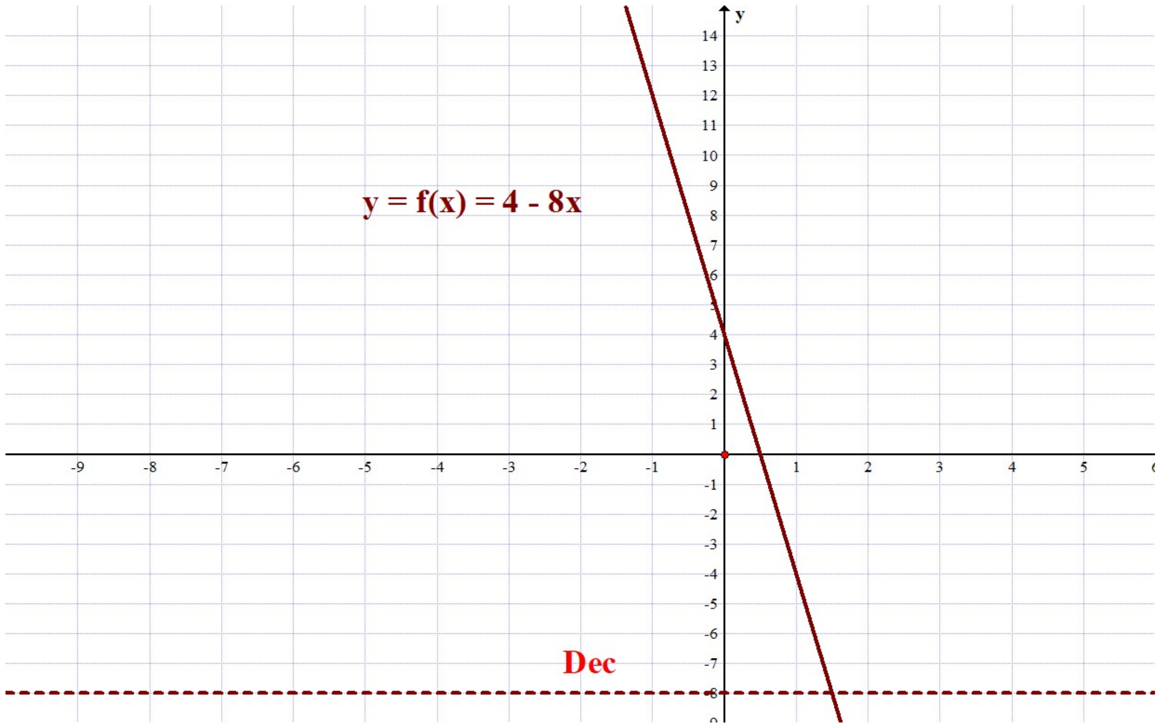
a. Relative Minimum Point: $(4, -12)$

b. Absolute Minimum Point: $(4, -12)$



3. $\mathbf{f}(\mathbf{x}) = 4 - 8\mathbf{x}$

There are no Maximum or Minimum Points.



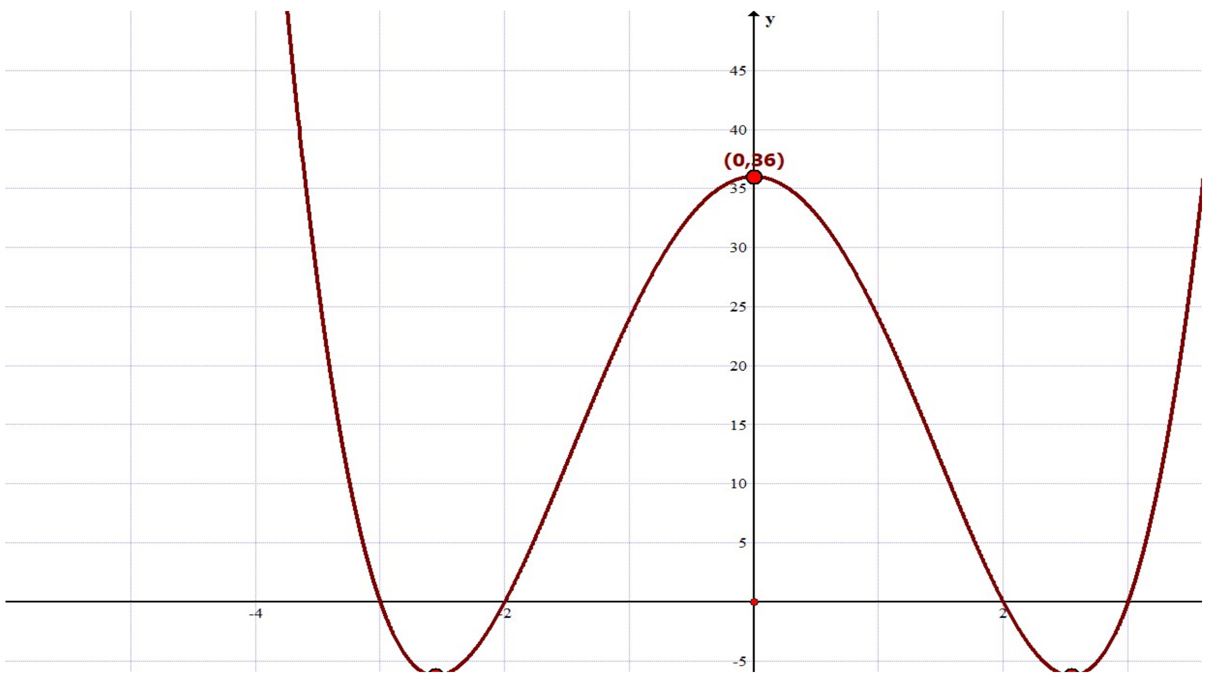
4. $\mathbf{f}(\mathbf{x}) = \mathbf{x}^4 - 13\mathbf{x}^2 + 36$

The graph yields

a. Relative Maximum Point: $(0,36)$

b. Relative Minimum Points: $\left(-\sqrt{\frac{13}{2}},-\frac{25}{4}\right);\left(\sqrt{\frac{13}{2}},-\frac{25}{4}\right)$

c. Absolute Minimum Points: $\left(-\sqrt{\frac{13}{2}},-\frac{25}{4}\right);\left(\sqrt{\frac{13}{2}},-\frac{25}{4}\right)$



5. $\mathbf{f}(\mathbf{x}) = \dfrac{2\mathbf{x}^3}{\mathbf{x}^2 - 4}$

Looking at the graph, we have

a. Relative Maximum Point: $\left(-2\sqrt{3}, -6\sqrt{3}\right)$

b. Relative Minimum Point: $\left(2\sqrt{3}, 6\sqrt{3}\right)$