# Linear Functions - More Topics

MATH by Wilson
Your Personal Mathematics Trainer
MathByWilson.com

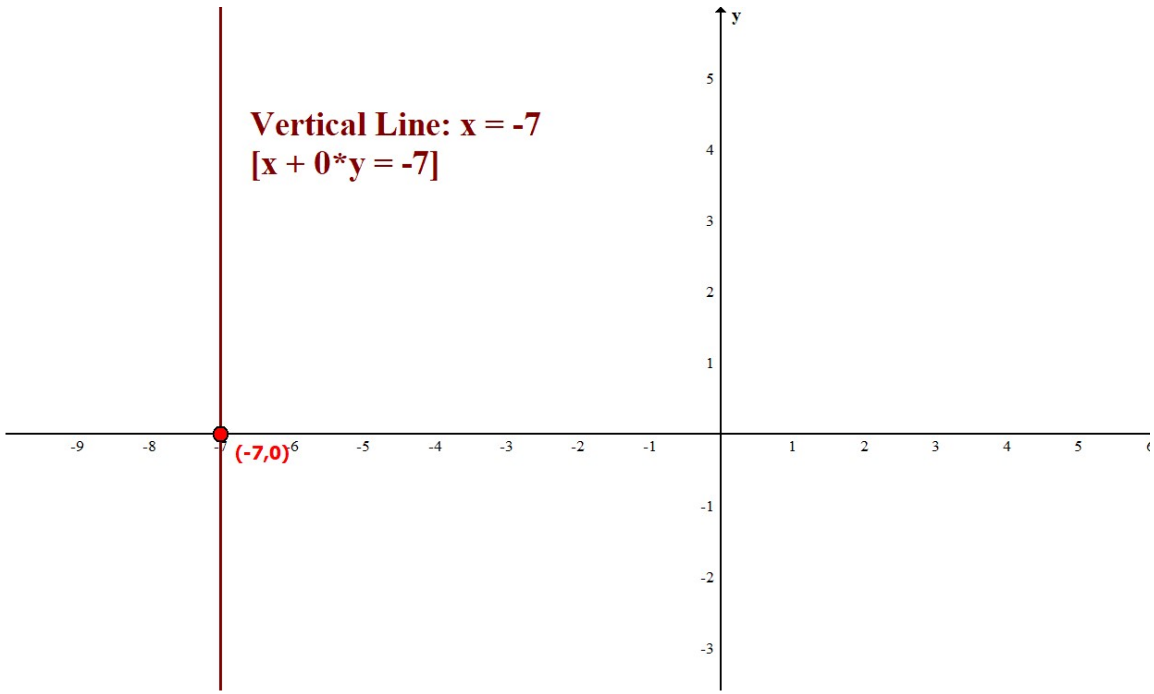
**Vertical Lines:** $x =$ Number

**Example 01:** Draw the graph of $x = -7$.

**Solution:** If we are in one dimension, $x = -7$ is just a point on the Horizontal Number Line:

If we are in two dimensions, $x = -7$ can be written as $x + 0 * y = -7$ so that points on its graph are $(-7, y)$ where y can be *any* number on the Vertical Number Line.

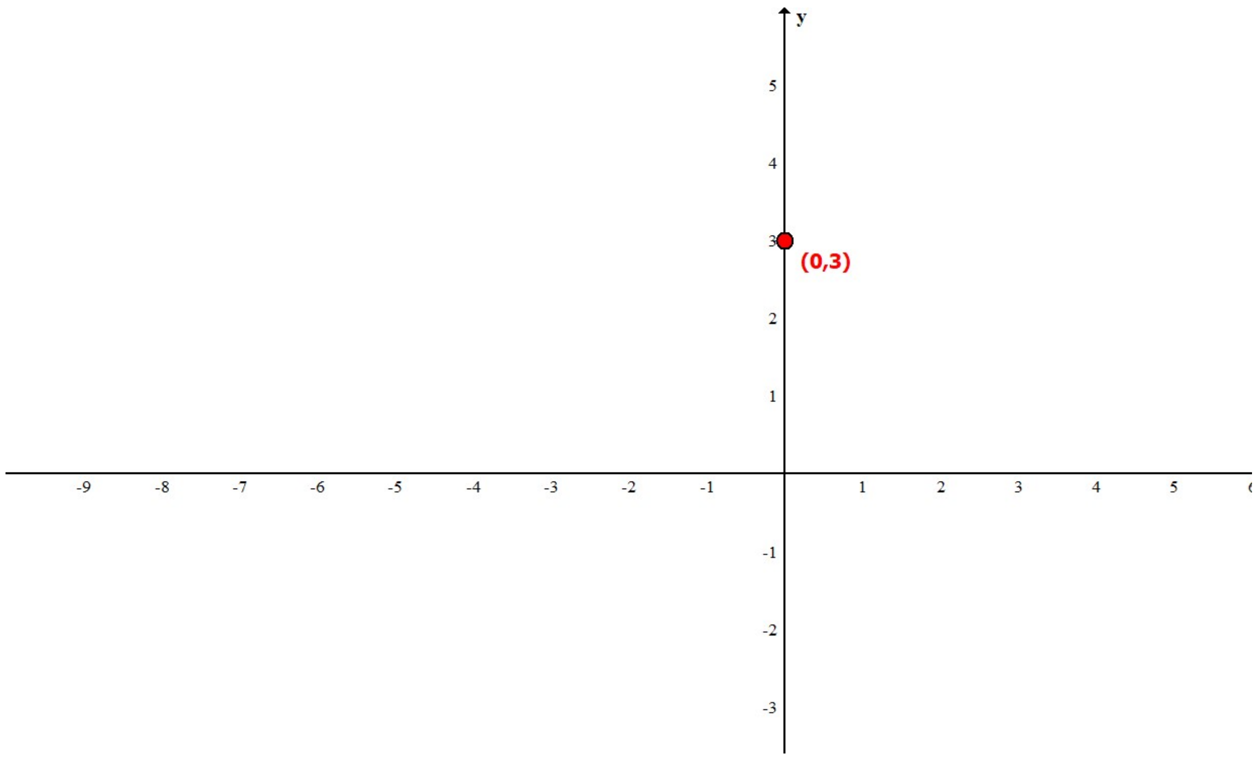


**Horizontal Lines:** $y =$ Number

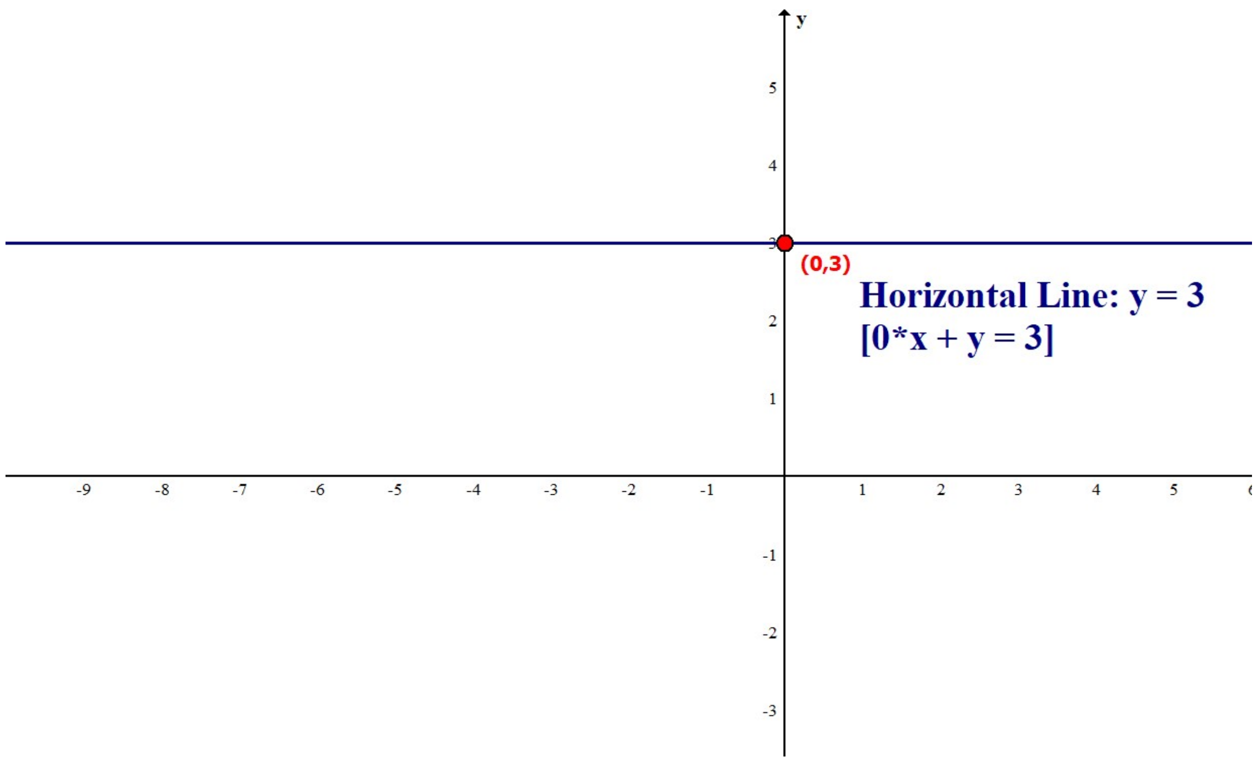
**Example 03:** Draw the graph of $y = 3$.

**Solution:** If we are in one dimension, $y = 3$ is just a point on the Vertical Number Line:




If we are in two dimensions, $y = 3$ can be written as $0 * x + y = 3$ so that points on its graph are $(x, 3)$ where x can be *any* number on the Horizontal Number Line.


y
5
4
3
2
1
-1
-2
-3
-9
-8
-7
-6
-5
-4
-3
-2
-1
1
2
3
4
5
(0,3)
Horizontal Line: y = 3
[0*x + y = 3]

**Parallel Lines:** $l_1 // l_2 \Rightarrow m_1 = m_2$ obviously

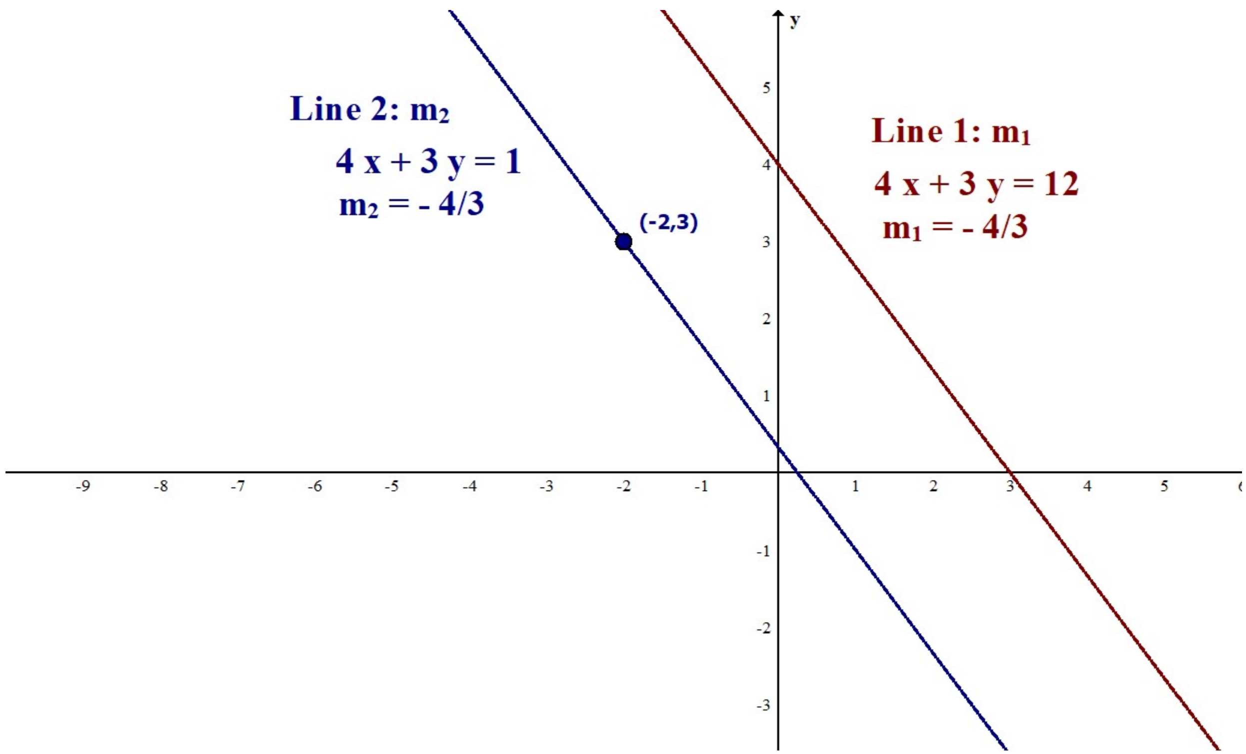
**Example 03:** Given $\ell_1 : 4x + 3y = 12$, find the equation of the line $\ell_2$ parallel to $\ell_1$ that passes through the point $P(x_2, y_2) = P(-2,3)$.

**Solution:** Solving $\ell_1 : 4x + 3y = 12$ for y yields $y = -\frac{4x}{3} + 4$ so that $m_2 = m_1 = -\frac{4}{3}$. Using the slope-intercept form $y = mx + b$, we obtain

$$
\begin{aligned}
y_2 &= m_2 \ x_2 + b \\
b &= y_2 - m_2 x_2 \\
&= 3 - \left( -\frac{4}{3}(-2) \right) \\
&= \frac{1}{3}
\end{aligned}
$$

Thus $\ell_2 : y = -\frac{4x}{3} + \frac{1}{3}$. The graphs are shown below:


Line 2: $m_2$
$4x + 3y = 1$
$m_2 = -4/3$
(-2,3)
Line 1: $m_1$
$4x + 3y = 12$
$m_1 = -4/3$
y

**Perpendicular Lines:** $l_1 \perp l_2 \Rightarrow m_1 * m_2 = -1$; this is NOT obvious but true: $m_2 = \frac{-1}{m_1} \quad (m_1 \neq 0)$

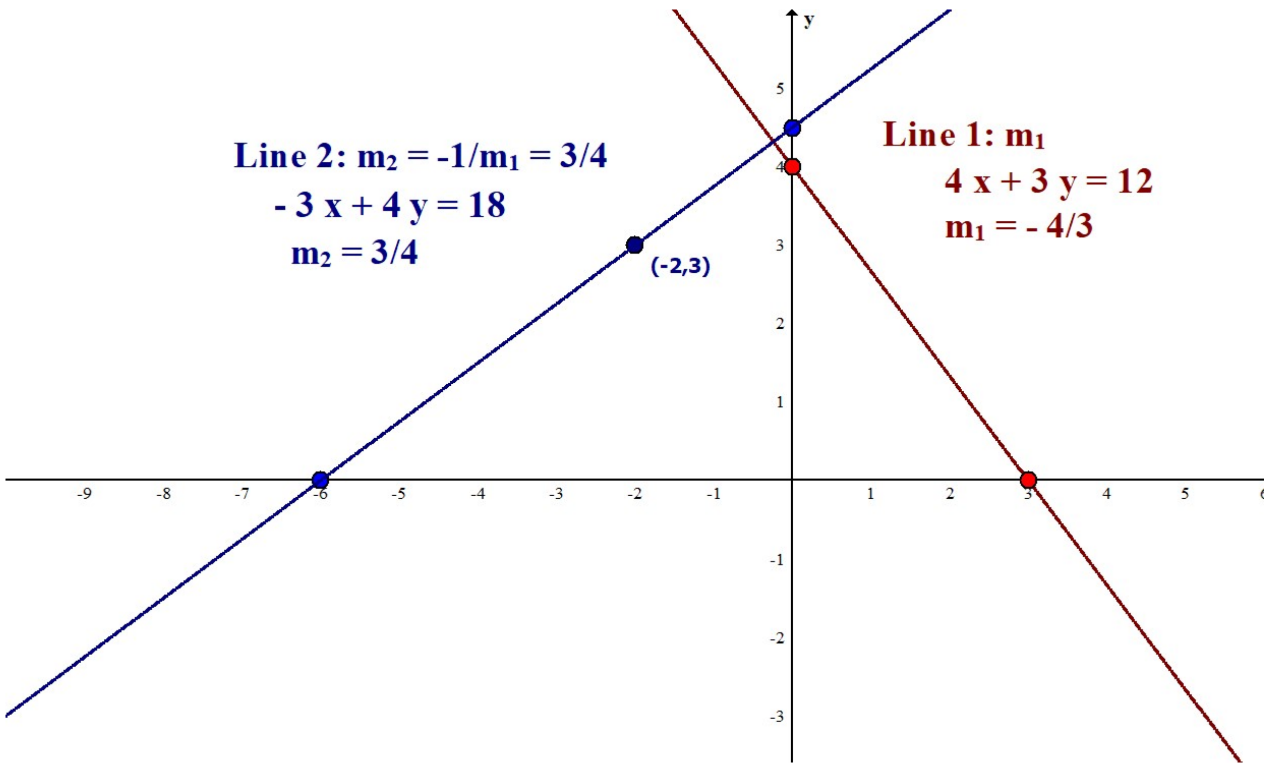
**Example 04:** Given $\ell_1 : 4x + 3y = 12$, find the equation of the line $\ell_2$ perpendicular to $\ell_1$ that passes through the point $P(x_2, y_2) = P(-2,3)$.

**Solution:** Solving $\ell_1 : 4x + 3y = 12$ for y yields $y = -\frac{4x}{3} + 4$ so that $m_2 = -\frac{1}{m_1} = \frac{3}{4}$. Using the slope-intercept form $y = mx + b$, we obtain

$$
\begin{aligned}
y_2 &= m_2 \; x_2 + b \\
b &= y_2 - m_2 x_2 \\
&= 3 - \left( \frac{3}{4}(-2) \right) \\
&= 9/2
\end{aligned}
$$

Thus $\ell_2 : y = \frac{3x}{4} + \frac{9}{2}$. The graphs are shown below:


y
5
Line 2: m₂ = -1/m₁ = 3/4
- 3 x + 4 y = 18
m₂ = 3/4
(-2,3)
Line 1: m₁
4 x + 3 y = 12
m₁ = - 4/3
4
3
2
1
-9
-8
-7
-6
-5
-4
-3
-2
-1
1
2
3
4
5
-1
-2
-3

**Average Rate of Change: With respect to an Interval $[a,b]$**

1. **Linear Functions – Straight Lines** $(y=f(x)=(\text{Slope})*x+(\text{y-intercept}))$

$$\frac{\text{Change in y values}}{\text{Change in x values}}=\frac{\Delta y}{\Delta x}=\frac{f(b)-f(a)}{b-a}=\text{Slope}$$

**Example 05:** Find the average rate of change for $y=f(x)=3x+4$ on $[2,5]$.

**Solution:**

We have

$$\frac{f(b)-f(a)}{b-a}=\frac{f(5)-f(2)}{5-2}=\frac{(15+4)-(6+4)}{3}$$
$$=\frac{9}{3}=3$$

2. **Other Functions – NOT Straight Lines** $(y=f(x)=\text{Formula})$

$$\frac{\text{Change in y values}}{\text{Change in x values}}=\frac{\Delta y}{\Delta x}=\frac{f(b)-f(a)}{b-a}$$

**Example 06:** Find the average rate of change for $y=f(x)=x^2-3x+4$ on $[1,3]$.

**Solution:**

We have

$$\frac{f(b)-f(a)}{b-a}=\frac{f(3)-f(1)}{3-1}=\frac{(9-9+4)-(1-3+4)}{2}$$

$$=\frac{4}{2}=2$$