

# FUNctions – Quadratic

$$y = f(x) = ax^2 + bx + c ; a \neq 0$$

## Five Point Method

$$\left[ \begin{array}{c} \text{MATH by Wilson} \\ \text{Your Personal Mathematics Trainer} \\ \text{MathByWilson.com} \end{array} \right]$$

Given

$$y = f(x) = 4x^2 + 8x - 2$$

$$a = 4$$

$$b = 8$$

$$c = -2$$

complete a **Five Point Method Template**:

a. **Domain**: Allowable x-values

$$\text{Domain} = \mathbb{R}_x = (-\infty, +\infty)_x$$

b. **POINTS #1 & #2**: Max of two x-intercept points

$$\text{Set } D = b^2 - 4ac$$

- (1)  $D < 0$ : NO x-intercept point
- (2)  $D = 0$ : ONE x-intercept point
- (3)  $D > 0$ : TWO x-intercept points

$$\text{x-intercepts POINTS(s): } \left( \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, 0 \right)$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-[8] \pm \sqrt{[8]^2 - 4[4][-2]}}{2[4]} = \frac{-8 \pm \sqrt{96}}{8}$$

$$= \frac{-8 \pm 4\sqrt{6}}{8} = -1 \pm \frac{\sqrt{6}}{2} \Rightarrow x \approx -2.2247, 0.22474$$

$$\Rightarrow \left( -1 - \frac{\sqrt{6}}{2}, 0 \right) \approx (-2.22, 0); \left( -1 + \frac{\sqrt{6}}{2}, 0 \right) \approx (0.22)$$

c. **POINT #3:**

**y-intercept POINT:**  $(0, c) = (0, -2)$

d. **POINT #4:**

**Symmetry POINT:**  $\left(-\frac{b}{a}, c\right)$  ; also called “cheap point”

$$\left(-\frac{b}{a}, c\right) = \left(-\frac{[8]}{4}, [-2]\right) = (-2, -2)$$

e. **POINT #5:**

**Vertex POINT:**  $\left(-\frac{b}{2a}, \frac{4ac - b^2}{4a}\right)$

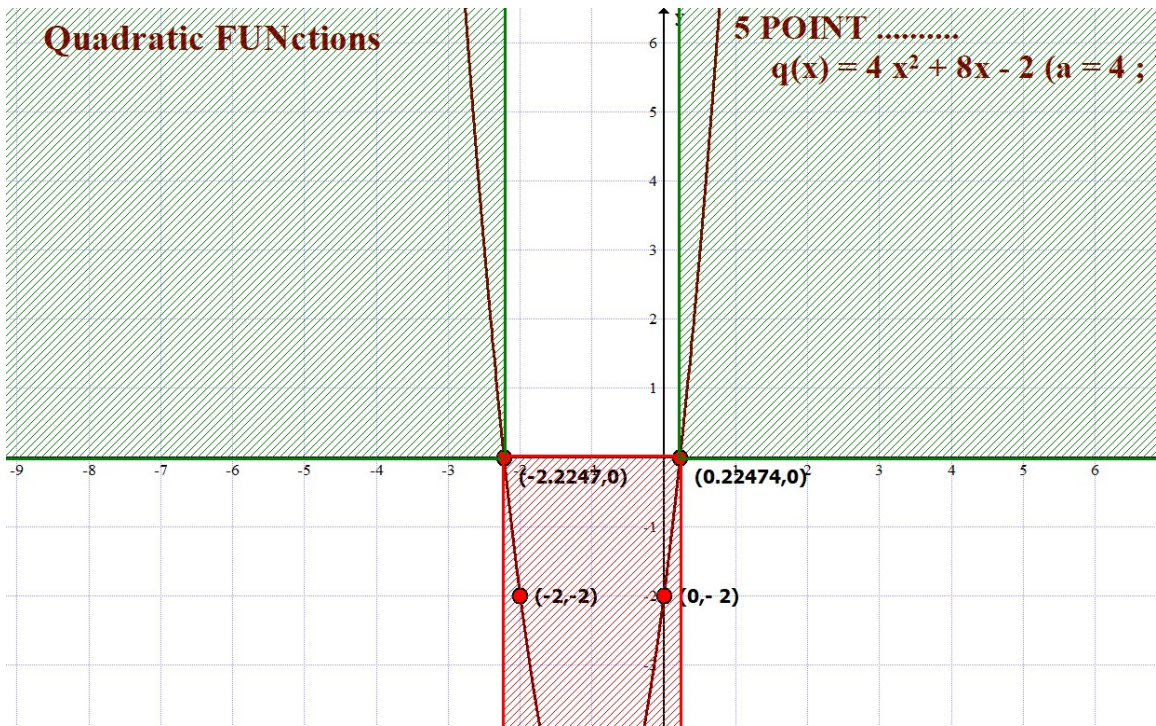
**Absolute Maximum point if  $a < 0$**

**Absolute Minimum point if  $a > 0$**

$a = 4 > 0 \Rightarrow$  Minimum

$$\Rightarrow \left(-\frac{b}{2a}, \frac{4ac - b^2}{4a}\right) = \left(-\frac{[8]}{2[4]}, \frac{4[4][-2] - [8]^2}{4[4]}\right) = (-1, -6)$$

f. **Graph:** PLOT the five (5) points (Max) and sketch the graph



g. **Positive:** The x-values where  $y > 0$

**Negative:** The x-values where  $y < 0$

Assuming two (2) x-intercept points, we have

(1) If  $a > 0$

$$\text{Positive} = \left( -\infty, \frac{-b - \sqrt{b^2 - 4ac}}{2a} \right] \cup \left[ \frac{-b + \sqrt{b^2 - 4ac}}{2a}, +\infty \right)$$

$$\text{Negative} = \left[ \frac{-b - \sqrt{b^2 - 4ac}}{2a}, \frac{-b + \sqrt{b^2 - 4ac}}{2a} \right]$$

$$\text{Neg } f = (-2.22, 0.22)_x$$

$$\text{Pos } f = (-\infty, -2.22)_x \cup (0.22, +\infty)_x$$

(2) If  $a < 0$

$$\text{Positive} = \left[ \frac{-b - \sqrt{b^2 - 4ac}}{2a}, \frac{-b + \sqrt{b^2 - 4ac}}{2a} \right]$$

$$\text{Negative} = \left( -\infty, \frac{-b - \sqrt{b^2 - 4ac}}{2a} \right] \cup \left[ \frac{-b + \sqrt{b^2 - 4ac}}{2a}, +\infty \right)$$

**h. Range:** Allowable y-values

$$\text{Range} = \left( -\infty, \frac{4ac - b^2}{4a} \right]_y \text{ if } a < 0$$

$$\text{Range} = \left[ \frac{4ac - b^2}{4a}, +\infty \right)_y \text{ if } a > 0$$

$$[-6, +\infty)_y$$