$\mathbf{h}(\mathbf{x})$ FUNction Summary TEMPLATE

Quadratic FUNction

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FUNCTION: $\mathbf{h}(\mathbf{x}) = 4(\mathbf{x}+1)^2 - 6 = 4\mathbf{x}^2 + 8\mathbf{x} - 2$

We first complete the Square:

$$\begin{aligned} 4\mathbf{x}^2 + 8\mathbf{x} - 2 &= 4\left(\mathbf{x}^2 + 2\mathbf{x} + [\ \ ]\right) - 2 + [\ \ ] \\ &= 4\left(\mathbf{x}^2 + 2\mathbf{x} + [1]\right) - 2 + [-4] \\ &= 4\left(\mathbf{x}+1\right)^2 - 6 \end{aligned}$$

$\mathbf{f}(\mathbf{x}) = \mathbf{x}^2$
$\mathbf{A} = 4$: Vertical Stretch
$\mathbf{B} = 1$: No effect
$\mathbf{C} = 1$: Horizontal Translation ; 1 unit to the left
$\mathbf{D} = -6$: Vertical Translation ; 6 units downward

Note: Since $\mathbf{h}(\mathbf{x})$ is "nice", we can find the graph of $\mathbf{h}(\mathbf{x})$ *before* finding **all** of the FUNction Summary Properties. However, we will still put its graph in Step #10 below. Appropriate calculations are shown at the bottom of the template.

1) DOMAIN:

Dom h $= \mathbb{R}_{\mathbf{x}}$

2) INTERCEPT POINT(S):

y-intercept point: $(0,-2)$; graph intersects to y-axis

$\mathbf{x}$ – intercept points: $\left(-1-\dfrac{\sqrt{6}}{2},0\right) \approx (-2.2247,0)$; $\left(-1+\dfrac{\sqrt{6}}{2},0\right) \approx (0.2247,0)$; graph intersect the x-axis twice

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3) CONTINUITY AND RELATED TOPICS:

CONT h $= \mathbb{R}_x$; there are NO breaks in the graph

DISCONT h $= \varnothing$; Empty Set

Hole h : N/A ; NO holes in the graph

Fin _ Jp h : N/A ; NO stair step behavior (finite jumps)

V _ Asy h : N/A ; NO vertical asymptotes

Advanced : N/A

POS h $= (-\infty, -2.22)_x \cup (0.22, +\infty)_x$; $h(x) > 0$

NEG h $= (-2.22, 0.22)_x$; $h(x) < 0$

4) BEHAVIOUR AT (TOWARD) INFINITY:

$\lim_{x \to -\infty} h(x) = +\infty$; as the x-values decrease without bound,
the cooresponding y-values increase without bound

$\lim_{x \to +\infty} h(x) = +\infty$; as the x-values increase without bound,
the cooresponding y-values increase without bound

H _ Asy h : N/A ; NO horizontal asymptotes

5) SYMMETRY (y -axis *or* $(0,0)$):

Even h : No ; graph NOT symmetric with respect to y-axis

Odd h : No ; graph NOT symmetric with respect to (0,0)

Other: $x = -1$; graph symmetric to vertical line x = - 1

6) INCREASING AND DECREASING:

INC h $= [-1, +\infty)_x$; graph going up on this interval

DEC h $= (-\infty, -1]$; graph going down on this interval

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7) RELATIVE MAXIMUM AND/OR MINIMUM POINT(S):

R_MAX_Pt h : N/A ; there is NO high point

R_MIN_Pt h : $(-1,-6)$; there is a low point

8) CONCAVITY:

CU h = $(-\infty,+\infty)_x$; the graph is ALWAYS smiling

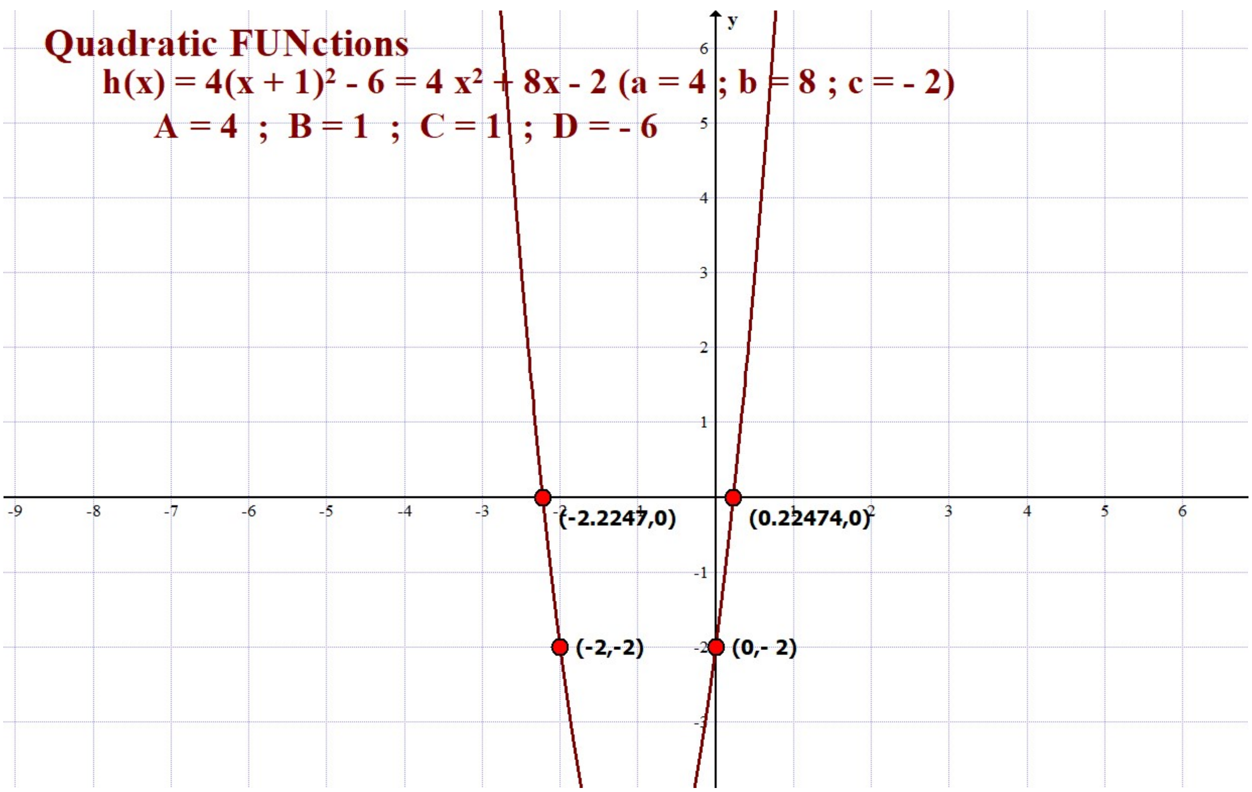
CD h = N/A ; the graph is NEVER frowning

9) INFLECTION POINT(S):

INF_Pt h : N/A ; there is NO change from smiling to frowning or vice versa

10) GRAPH:

GRAPH h :



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11) ABSOLUTE MAXIMUM AND/OR MINIMUM POINT(S):

$\mathbf{A\_MAX\_Pt\ h}$: N/A ; there is NO highest point

$\mathbf{A\_MIN\_Pt\ h}$: $(-1,-6)$; there is a lowest point

12) RANGE:

$\mathbf{RANGE\ h} = [-6,+\infty)_{\mathbf{y}}$; the allowable y values

Calculations:

1. Intercepts:
   a. y-intercept: $\mathbf{h}(0) = -2 \Rightarrow (0,-2)$
   b. x-intercepts: $\mathbf{h}(\mathbf{x}) \overset{\mathbf{SET}}{=} 0 \Rightarrow$

$$4(\mathbf{x}+1)^2 - 6 = 0 \Rightarrow (\mathbf{x}+1)^2 = \frac{6}{4} \Rightarrow \mathbf{x}+1 = \pm\sqrt{\frac{6}{4}}$$

$$\Rightarrow \mathbf{x} = -1 \pm \frac{\sqrt{6}}{2} \Rightarrow \left(-1-\frac{\sqrt{6}}{2},0\right);\left(-1+\frac{\sqrt{6}}{2},0\right)$$

$$\Rightarrow \approx (-2.22,0);(0.22)$$

2. Continuity:

$$\overbrace{\underline{\qquad\qquad\qquad}}^{+}\,-2.22\,\overbrace{\underline{\qquad\qquad\qquad}}^{-}\,0.22\,\overbrace{\underline{\qquad\qquad\qquad}}^{+}$$

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