Real Number System

(Equality and Inequality Properties)

Our current goal is to list the equality and inequality properties the real numbers possess.

We have encountered various properties the set of real numbers possesses as we considered its type, operations, … Now, let's list and name these properties that involve equality (=) in one place.

Assume that a, b, and c are real numbers:

Equality Properties:

Real Number Properties	Comments	Examples
1. Closure		
a. a + b is a real number	The *sum* of two real numbers is a real number	2 + 5 = 7 is a real number
b. a*b (= a b) is a real number	The *product* of two real numbers is a real number	2*5 = 10 is a real number
2. Commutative Property		

a. a + b = b + a	Two real numbers can be *added* in either order	2 + 3 = 5 = 3 + 2
b. a*b = b*a	Two real numbers can be *multiplied* in either order	2*7 = 14 = 7*2
	Note: a – b is probably not b - a	5 – 2 = 3 2 – 5 = - 3
3. Associative Property		
a. (a + b) + c = a + (b + c)	The *sum* of three real numbers does not depend on the order they are *added*.	(4 + 3) + 5 = 7 + 5 = 12 4 + (3 + 5) = 4 + 8 = 12
b. a*(b*c) = (a*b)*c	The *product* of three real numbers does not depend on the order they are *multiplied*.	2*(4*3) = 2*12 = 24 (2*4)*3 = 8*3 = 24
4. Identity Property		
a. a + 0 = a = 0 + a	"0" is called the ***additive identity***. The *sum* of a real number and "0" is the number itself.	6 + 0 = 6 = 0 + 6
b. a*1 = a = 1*a	"1" is called the **multiplicative identity**. The *product* of a real number and "1" is the number itself	9*1 = 9 = 1*9
5. Inverse Properties		

a. For each real number "a", there is a real number "- a" such that a + (- a) = 0 = (- a) + a	"- a" is called the **additive inverse** of "a". The *sum* of a real number and its additive inverse is "0" Note: a + (- a) is frequently written a - a	4 + (-4) = 0 = (-4) + 4
b. For each real number a (not equal to 0), there is a real number "1/a" such that a*(1/a) = 1 = (1/a)*a	(1/a) is called the **multiplicative inverse** of "a". The *product* of a real number and its multiplicative inverse is "1"	8*(1/8) = 1 = (1/8)*8
6. Distributive Property		
a. a*(b + c) = a*b + a*c	The *product* of a real number and the *sum* of two real numbers is the *sum* of the *product* of the first number and each of the other numbers.	3*(4 + 2) = 3*6 = 18 3*4 + 3*2 = 12 + 6 =18
b. a*(b – c) = a*b – a*c	The *product* of a real number and the *difference* of two real numbers is the *difference*	3*(4 - 2) = 3*2 = 6 3*4 - 3*2 = 12 - 6 = 6

	of the *product* of the first number and each of the other numbers.	
7. Multiplicative Property of Zero		
a*0 = 0*a = 0	The *product* of a real number and “0” is “0”	2*0 = 0 = 0*2

Inequality Properties:

Terms ("+" or "-")		
Addition	1. If $a \begin{Bmatrix} \leq \\ < \\ \geq \\ > \end{Bmatrix} b$ then $a+c \begin{Bmatrix} \leq \\ < \\ \geq \\ > \end{Bmatrix} b+c$	$6 \leq 8$ implies $6+3=9 \leq 11=8+3$
Subtraction	2. If $a \begin{Bmatrix} \leq \\ < \\ \geq \\ > \end{Bmatrix} b$ then $a-c \begin{Bmatrix} \leq \\ < \\ \geq \\ > \end{Bmatrix} b-c$	$6 \leq 8$ implies $6-3=3 \leq 5=8-3$

Factors ("*" or "/")		
	1. If c > 0 then a. $a \left\{\begin{matrix} \le \\ < \\ \ge \\ > \end{matrix}\right\} b$ then $a*c \left\{\begin{matrix} \le \\ < \\ \ge \\ > \end{matrix}\right\} b*c$	$3 \le 7$ implies $3*2 = 6 \le 14 = 7*2$
	b. $a \left\{\begin{matrix} \le \\ < \\ \ge \\ > \end{matrix}\right\} b$ then $\frac{a}{c} \left\{\begin{matrix} \le \\ < \\ \ge \\ > \end{matrix}\right\} \frac{b}{c}$	$3 \le 7$ implies $\frac{3}{2} \le \frac{7}{2}$

	2. If c < 0 then	
	a. $a\left\{\begin{matrix}\le\\<\\\ge\\>\end{matrix}\right\}b$ then $a*c\left\{\begin{matrix}\ge\\>\\\le\\<\end{matrix}\right\}b*c$	$3\le 8$ implies $3*(-2)=-6\ge -16=8*(-2)$
	b. $a\left\{\begin{matrix}\le\\<\\\ge\\>\end{matrix}\right\}b$ then $\frac{a}{c}\left\{\begin{matrix}\ge\\>\\\le\\<\end{matrix}\right\}\frac{b}{c}$	$3\le 8$ implies $3/(-2)=-\frac{3}{2}\ge -4=8/(-2)$

WARNING: When multiplying or dividing by a negative real number, the direction ("sense") of the inequality switches; for example $"\le"$ changes to $"\ge"$. **PLEASE Be Careful!**