FUNction Summary Definitions
("Nice Real-valued" FUNctions)

FUNction Definition: $\mathbf{y} = \mathbf{f}(\mathbf{x}) =$ Formula

1) **DOMAIN: Dom f** = ?
    a) Allowable x values – inputs – independent variables
    b) Maximum – ALL real numbers (Horizontal): Frequently - Unions of x-axis intervals
    c) Given or $\{\mathbf{x} \in \mathbb{R} \mid \mathbf{f}(\mathbf{x}) \in \mathbb{R}\}$ - Real-valued functions
        a. Cannot divide by zero
        b. Cannot have a negative under an even root
    d) Projection of graph onto the x-axis
2) **INTERCEPT POINT(S):**
    a) y-Intercept POINT: $\mathbf{Int_y\_Pt\ f} = ?$ Max of one ; Action verb: Evaluate (Arithmetic)
        (1) $(0, \mathbf{f}(0))$ if $0 \in \mathbf{Dom\ f}$
        (2) Where graph intersects the y-axis
    b) x-Intercept POINT(s): $\mathbf{Int_x\_Pt\ f} = ?$ Action verb: Solve (Equation)
        (1) $\{\mathbf{x} \in \mathbf{Dom\ f} \mid \mathbf{y} = \mathbf{f}(\mathbf{x}) = 0\} \Rightarrow (\mathbf{x}_0, \mathbf{f}(\mathbf{x}_0))$ where $\mathbf{f}(\mathbf{x}_0) = 0$
        (2) Where graph intersects the x-axis
3) **CONTINUITY AND RELATED TOPICS:**
    a) Continuity: **Cont f** = ?
        (1) No breaks in graph
        (2) Frequently – Union of x-axis intervals
        (3) Graph connected
    b) Discontinuity: **Discont f** = ?
        (1) Breaks
            a. Hole
            b. Vertical Asymptote
            c. Finite Jump
            d. Undefined regions – NO graph

(2) Points & x-axis intervals

(3) Graph disconnected

c) Positive/Negative Nature

(1) **Pos f** $= ?$

a. $\{\mathbf{x} \in \mathbf{Dom\ f} \,|\, \mathbf{f}(\mathbf{x}) > 0\}$

b. Where graph is *above* the x-axis: Frequently - Union of x-axis intervals

(2) **Neg f** $= ?$

a. $\{\mathbf{x} \in \mathbf{Dom\ f} \,|\, \mathbf{f}(\mathbf{x}) < 0\}$

b. Where graph is *below* x-axis: Frequently - Union of x-axis intervals

4) BEHAVIOUR AT (TOWARD) INFINITY:

a) $\mathbf{x} \to +\infty \Rightarrow \mathbf{f}(\mathbf{x}) \to ?$ (Calculus: $\mathbf{\lim\limits_{x \to +\infty} f}(\mathbf{x}) = ?$)

(1) As the x values are chosen *larger and larger* (increase without bound), what the corresponding f(x) values are doing. (Left to Right)

(2) What the graph looks like as the x values increase without bound

b) $\mathbf{x} \to -\infty \Rightarrow \mathbf{f}(\mathbf{x}) \to ?$ (Calculus: $\mathbf{\lim\limits_{x \to -\infty} f}(\mathbf{x}) = ?$)

(1) As the x values are chosen *smaller and smaller* (decrease without bound), what the corresponding f(x) values are doing. (Right to Left)

(2) What the graph looks like as the x values decrease without bound

5) SYMMETRY ($\mathbf{y}$ -axis *or* $(0,0)$):

a) **Even f** $= ?$

(1) $\mathbf{f}(-\mathbf{x}) = \ldots = \mathbf{f}(\mathbf{x})$

(2) Graph is symmetric with respect to the y-axis

b) **Odd f** $= ?$

(1) $\mathbf{f}(-\mathbf{x}) = \ldots = -\mathbf{f}(\mathbf{x})$

(2) Graph is symmetric with respect to the $(0,0)$

6) **INCREASING & DECREASING:**

a) **Inc f** = ?

(1) As x values increase (Left to Right), $\mathbf{f}(\mathbf{x})$ values increase

(2) Union of x axis intervals where $\mathbf{f}(\mathbf{x})$ values increase: $\mathbf{x}_1 < \mathbf{x}_2 \Rightarrow \mathbf{f}(\mathbf{x}_1) \le \mathbf{f}(\mathbf{x}_2)$

(3) Graph going up (Left to Right)

Calculus: $\mathbf{f}'(\mathbf{x}) > 0$ on an interval $\mathbf{I}$

b) **Dec f** = ?

(1) As x values increase (Left to Right), $\mathbf{f}(\mathbf{x})$ values decrease

(2) Union of x axis intervals where $\mathbf{f}(\mathbf{x})$ values decrease: $\mathbf{x}_1 < \mathbf{x}_2 \Rightarrow \mathbf{f}(\mathbf{x}_1) \ge \mathbf{f}(\mathbf{x}_2)$

(3) Graph going down (Left to Right)

Calculus: $\mathbf{f}'(\mathbf{x}) < 0$ on an interval $\mathbf{I}$

7) **RELATIVE MAXIMUM/MINIMUM POINTS:**

a) Relative Maximum Point(s):

(1) Relative High Point(s)

(2) $\left(\mathbf{x}_{\mathbf{RMax}}, \mathbf{f}\left(\mathbf{x}_{\mathbf{RMax}}\right)\right)$ where $\mathbf{f}(\mathbf{x}) \le \mathbf{f}\left(\mathbf{x}_{\mathbf{RMax}}\right)$ for $\mathbf{x}$ "close" to $\mathbf{x}_{\mathbf{RMax}}$

Calculus: Possible: $\left(\mathbf{x}_0, \mathbf{f}\left(\mathbf{x}_0\right)\right)$ where $\mathbf{f}'(\mathbf{x}_0) = 0$ **or undefined**

Actual: $\mathbf{f}'$ *increasing* to the left of $\mathbf{x}_0$ and *decreasing* to the right

b) Relative Minimum Point(s)

(1) Relative Low Point(s)

(2) $\left(\mathbf{x_{RMin}}, \mathbf{f}\left(\mathbf{x_{RMin}}\right)\right)$ where $\mathbf{f}(\mathbf{x}) \geq \mathbf{f}\left(\mathbf{x_{RMin}}\right)$ for $\mathbf{x}$ "close" to $\mathbf{x_{RMin}}$

Calculus: Possible: $\left(\mathbf{x}_0, \mathbf{f}\left(\mathbf{x}_0\right)\right)$ where $\mathbf{f}'(\mathbf{x}_0) = 0$ **or undefined** Actual: $\mathbf{f}'$ *decreasing* to the left of $\mathbf{x}_0$ and *increasing* to the right

8) **CONCAVE UPWARD & DOWNWARD**

a) Concave Upward:

(1) Union of x axis intervals where graph is "smiling"

(2) Tangent lines below graph

Calculus: $\mathbf{f}''(\mathbf{x}) > 0$ on an interval $\mathbf{I}$

b) Concave Downward:

(1) Union of x axis intervals where graph is "frowning"

(2) Tangent lines above graph

Calculus: $\mathbf{f}''(\mathbf{x}) < 0$ on an interval $\mathbf{I}$

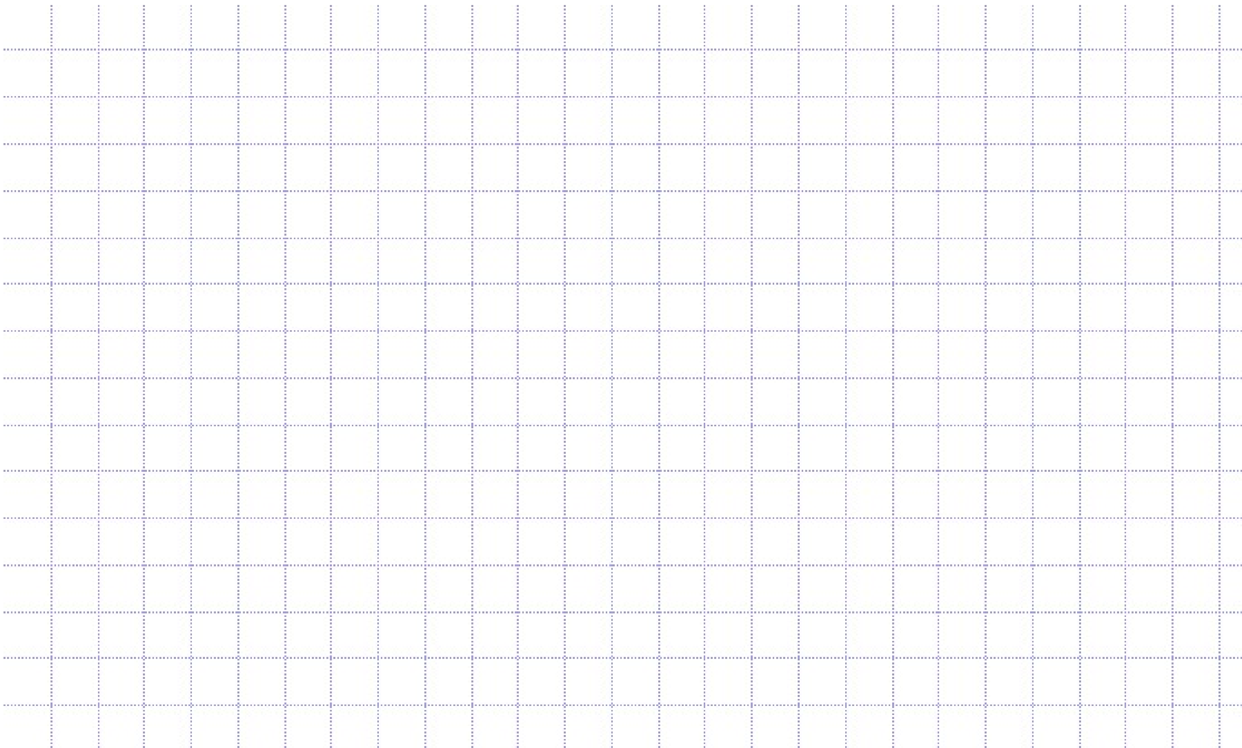
9) **INFLECTION POINTS:**

a) Points where graph changes from concave upward to downward

b) Points where graph changes from concave downward to upward

Calculus: Possible: $\left(\mathbf{x}_0, \mathbf{f}\left(\mathbf{x}_0\right)\right)$ where $\mathbf{f}''(\mathbf{x}_0) = 0$ **or undefined** Actual: $\mathbf{f}$ is *concave upward* on one side of $\mathbf{x}_0$ and *concave downward* on the other

10) **GRAPH f** :



11) **ABSOLUTE MAXIMUM & MINIMUM POINTS:**

a) Absolute Maximum Point(s):

(1) Absolute High Point(s)

(2) $\left(\mathbf{x}_{\mathbf{AMax}}, \mathbf{f}\left(\mathbf{x}_{\mathbf{AMax}}\right)\right)$ where $\mathbf{f}(\mathbf{x}) \leq \mathbf{f}\left(\mathbf{x}_{\mathbf{AMax}}\right)$ for $\mathbf{x} \in \mathbf{Dom\ f}$

b) Absolute Minimum Point(s)

(1) Absolute Low Point(s)

(2) $\left(\mathbf{x}_{\mathbf{AMin}}, \mathbf{f}\left(\mathbf{x}_{\mathbf{AMin}}\right)\right)$ where $\mathbf{f}(\mathbf{x}) \geq \mathbf{f}\left(\mathbf{x}_{\mathbf{AMin}}\right)$ for $\mathbf{x} \in \mathbf{Dom\ f}$

12) **RANGE: Range f** = ?

a) Allowable y values – outputs – dependent variables

b) Maximum – All real numbers (Vertical): Frequently - Union of y-axis intervals

c) Given or $\left\{\mathbf{y} = \mathbf{f}(\mathbf{x}) \in \mathbb{R} \middle| \mathbf{x} \in \mathbf{Dom\ f}\right\}$

d) Projection of graph onto the y-axis

e) Solve $\mathbf{y} = \mathbf{f}(\mathbf{x})$ for $\mathbf{x}$ if possible

a. Cannot divide by zero
b. Cannot have a negative under an even root

