

Systems of Linear Equations
(2 by 2: “2” equations with “2” unknowns)

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Systems: More than 1 equation ; more than 1 unknown

Linear: Powers of the unknowns is 1: $x^1 = x, y^1 = y, \dots$

A Linear System may have one solution!

- 1. 2x2:** Finding the intercept points of the two (2) lines to help us determine the geometrical solution:

$$l_1 : 5x - 2y = 19$$

$$l_2 : 3x + 4y = 1$$

$$\text{Solution : } (x, y) = (?, ?)$$

l_1 :

$$\text{x-int: Set } y = 0 \Rightarrow 5x = 19 \Rightarrow x = \frac{19}{5} \Rightarrow \left(\frac{19}{5}, 0\right)$$

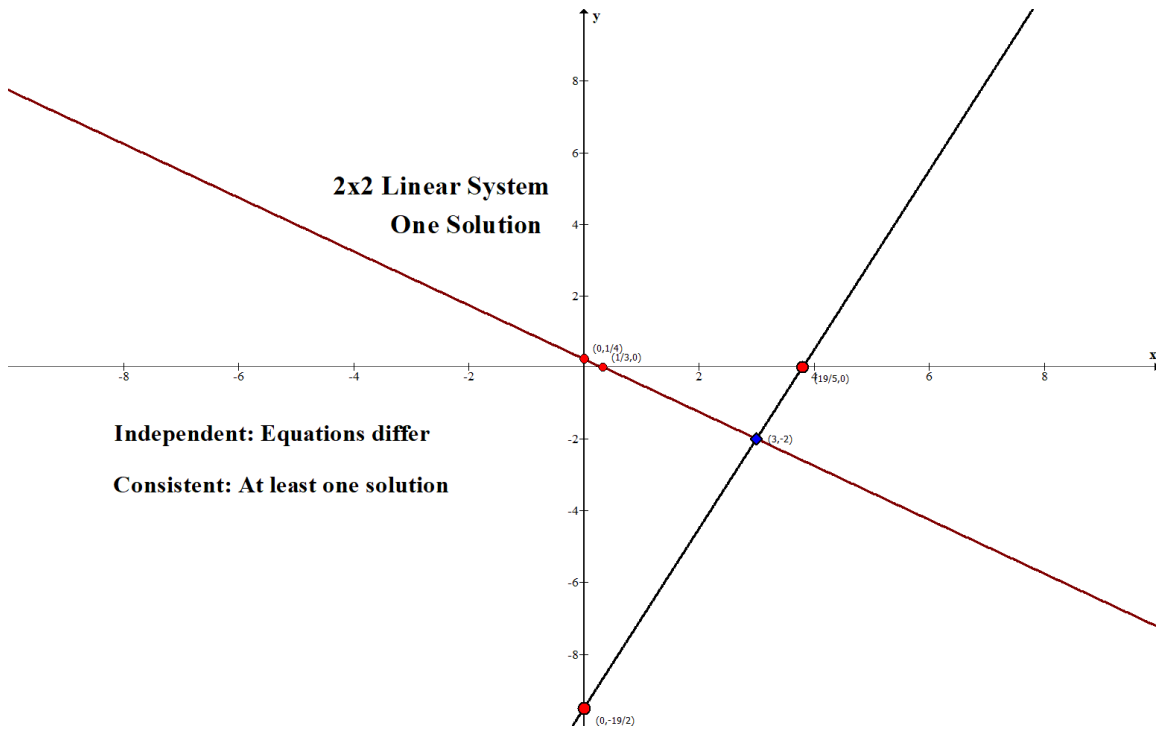
$$\text{y-int: Set } x = 0 \Rightarrow -2y = 19 \Rightarrow y = -\frac{19}{2} \Rightarrow \left(0, -\frac{19}{2}\right)$$

l_2 :

$$\text{x-int: Set } y = 0 \Rightarrow 3x = 1 \Rightarrow x = \frac{1}{3} \Rightarrow \left(\frac{1}{3}, 0\right)$$

$$\text{y-int: Set } x = 0 \Rightarrow 4y = 1 \Rightarrow y = \frac{1}{4} \Rightarrow \left(0, \frac{1}{4}\right)$$

a. Graphical Method: Straight Lines – 2 dimensional



b. Substitution Method

$$l_1 : 5x - 2y = 19$$

$$l_2 : 3x + 4y = 1$$

2 equations with 2 unknowns \Rightarrow with Substitution \Rightarrow 1 equation with 1 unknown

Substitute x for y stuff **OR** substitute y for x stuff.

***** Substitution: Solve l_2 for x (One of four choices)

$$3x + 4y = 1 \Rightarrow 3x = 1 - 4y \Rightarrow x = \frac{1 - 4y}{3}$$

$$5x - 2y = 19 \Rightarrow 5\left(\frac{1 - 4y}{3}\right) - 2y = 19 \text{ 1 Equation with 1 unknown}$$

$$5\left(\frac{1 - 4y}{3}\right) - 2y = 19 \Rightarrow \text{Multiply both sides by 3 (MUST BE CAREFUL!)}$$

to clear the fractions

$$3\left[5\left(\frac{1 - 4y}{3}\right) - 2y\right] = 3[19] \Rightarrow 5 - 20y - 6y = 57 \Rightarrow -26y = 52 \Rightarrow y = -2$$

Substitute for y in l_1

$$\Rightarrow 5x - 2y = 19 \Rightarrow 5x - 2(-2) = 19 \Rightarrow 5x + 4 = 19 \Rightarrow 5x = 15 \Rightarrow x = 3$$

Solution: $(3, -2)$

c. Elimination (Addition/Subtraction) Method

$$l_1 : 5x - 2y = 19$$

$$l_2 : 3x + 4y = 1$$

2 equations with 2 unknowns \Rightarrow with Elimination \Rightarrow 1 equation with 1 unknown

*****Eliminate y (Two choices)

$$2 * l_1 + l_2 :$$

$$2 * (5x - 2y = 19) \Rightarrow$$

$$10x - 4y = 38$$

$$\underline{3x + 4y = 1} \text{ ADD}$$

$$13x = 39 \Rightarrow x = 3$$

Substitute for x in l_2 :

$$3x + 4y = 1 \Rightarrow 3(3) + 4y = 1 \Rightarrow 4y = 1 - 9 = -8 \Rightarrow y = -2$$

Solution : $(3, -2)$

*****Eliminate x (Two choices)

$$l_1 : 5x - 2y = 19$$

$$l_2 : 3x + 4y = 1$$

$$3 * l_1 - 5 * l_2 :$$

$$15x - 6y = 57$$

$$-15x - 20y = -5$$

$$\text{ADD: } 0x - 26y = 52 \Rightarrow y = \frac{52}{-26} = -2$$

Substitute for y in l_1 : $5x - 2(-2) = 19 \Rightarrow 5x + 4 = 19 \Rightarrow 5x = 15 \Rightarrow x = 3$

Solution : $(3, -2)$

d. Matrix Methods

$$l_1 : 5x - 2y = 19$$

$$l_2 : 3x + 4y = 1$$

$x \quad y$

$$\left[\begin{array}{cc|c} 5 & -2 & 19 \\ 3 & 4 & 1 \end{array} \right] \quad \text{Goal: } \left[\begin{array}{cc|c} 1 & 0 & x \text{ sol} \\ 0 & 1 & y \text{ sol} \end{array} \right]$$

$$\frac{1}{5} * R_1 \rightarrow R_1 : \left[\begin{array}{cc|c} 1 & \frac{-2}{5} & \frac{19}{5} \\ 3 & 4 & 1 \end{array} \right]$$

$$-3 * R_1 + R_2 \rightarrow R_2 : \left[\begin{array}{cc|c} 1 & \frac{-2}{5} & \frac{19}{5} \\ 0 & \frac{26}{5} & \frac{-52}{5} \end{array} \right]$$

$$\frac{5}{26} * R_2 \rightarrow R_2 : \left[\begin{array}{cc|c} 1 & \frac{-2}{5} & \frac{19}{5} \\ 0 & 1 & -2 \end{array} \right]$$

$$\frac{2}{5} * R_2 + R_1 \rightarrow R_1 : \left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & -2 \end{array} \right]$$

Solution : (3, -2)

e. Determinants – see other notes

f. Matrix Inverse – see other notes

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