Systems of Linear Equations

(3 by 3: "3" equations with "3" unknowns)

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Systems: More than 1 equation & more than 1 unknown $(x, y, z, \ldots)$

Linear: Powers of the unknowns is 1 $\left(x = x^1, y = y^1, \ldots\right)$

1. **3x3: Trade 3x3 for 2x2 for 1x1 & "Back Substitute" if necessary**

$$p_1 : x - 2y + 3z = 14$$
$$p_2 : 2x + 3y - z = -7$$
$$p_3 : 4x + y + 2z = 7$$
$$Solution : (x, y, z) = (?,?,?)$$

   a. **Graphing: 3 Planes – 3 dimensional – Will NOT do**

b. Substitution Method:

$p_1 : x - 2y + 3z = 14$
$p_2 : 2x + 3y - z = -7$
$p_3 : 4x + y + 2z = 7$

3x3 System $\Rightarrow$ Trade for 2x2 System:
Smart to choose p_1 for x OR p_2 for z $\Rightarrow$ NO fractions ...

Solve p_1 for x:
$x - 2y + 3z = 14 \Rightarrow x = 14 + 2y - 3z$

Substitute for x in p_2 & p_3 Note: Now p_2 & p_3 have ONLY y & z in them

$***p_2$
$2x + 3y - z = -7 \Rightarrow 2(14 + 2y - 3z) + 3y - z = -7 \Rightarrow 28 + 4y - 6z + 3y - z = -7$
$\Rightarrow 7y - 7z = -35 \Rightarrow y - z = -5$

$***p_3$
$4x + y + 2z = 7 \Rightarrow 4(14 + 2y - 3z) + y + 2z = 7 \Rightarrow 56 + 8y - 12z + y + 2z = 7$
$\Rightarrow 9y - 10z = -49$

$$\therefore \begin{cases} p_2': y - z = -5 \quad (\text{or } y = z - 5) \\ p_3': 9y - 10z = -49 \end{cases}$$

$2x2$ System $\Rightarrow$ Trade for 1x1 System:

Solve p_2' for y & substitute in $p_3' \Rightarrow 1x1$ (z ONLY):

$9y - 10z = -49 \Rightarrow 9(z-5) - 10z = -49 \Rightarrow 9z - 45 - 10z = -49 \Rightarrow -z = -4$

$\Rightarrow z = 4 \Rightarrow (x, y, 4)$

Back Substitute: Substitute for z in p_2':

$y - z = -5 \Rightarrow y - (4) = -5$

$\Rightarrow y = -1 \Rightarrow (x, -1, 4)$

Back Substitute in $x = 14 + 2y - 3z \Rightarrow x = 14 + 2(-1) - 3(4) \Rightarrow x = 0$

Solution: $(x, y, z) = (0, -1, 4)$

c. Elimination (Addition/Subtraction) Method:

$p_1 : x - 2y + 3z = 14$
$p_2 : 2x + 3y - z = -7$
$p_3 : 4x + y + 2z = 7$

Eliminate x in p_1 :

p_1 & $p_2 : -2 * p_1 + p_2$

$$\begin{array}{rl} -2x + 4y - 6z = -28 & \\ \underline{2x + 3y - \ \ z = -7} & \text{Add} \\ 7y - 7z = -35 & \left(p_2' : y - z = -5\right) \end{array}$$

p_1 & $p_3 : -4 * p_1 + p_3$

$$\begin{array}{rl} -4x + 8y - 12z = -56 & \\ \underline{4x + y + \ 2z = 7} & \text{Add} \\ 9y - 10z = -49 & \left(p_3' : 9y - 10z = -49\right) \end{array}$$

$$\therefore \begin{cases} p_2' : y - z = -5 & (y = z - 5) \\ p_3' : 9y - 10z = -49 \end{cases}$$

Previously solved: $(x, y, z) = (0, -1, 4)$

d. Matrix Method:

$p_1 : x - 2y + 3z = 14$
$p_2 : 2x + 3y - z = -7$
$p_3 : 4x + y + 2z = 7$

$$\begin{array}{ccc} x & y & z \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 14 \\ 2 & 3 & -1 & -7 \\ 4 & 1 & 2 & 7 \end{array}\right] \qquad \text{Goal: } \left[\begin{array}{ccc|c} 1 & 0 & 0 & x\ sol \\ 0 & 1 & 0 & y\ sol \\ 0 & 0 & 1 & z\ sol \end{array}\right]$$

$-2 * R_1 + R_2 \to R_2$
$-4 * R_1 + R_3 \to R_3$

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 14 \\ 0 & 7 & -7 & -35 \\ 0 & 9 & -10 & -49 \end{array}\right]$$

$\frac{1}{7} * R_2 \to R_2$

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 14 \\ 0 & 1 & -1 & -5 \\ 0 & 9 & -10 & -49 \end{array}\right]$$

$\frac{1}{7}*R_2 \to R_2$

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 14 \\ 0 & 1 & -1 & -5 \\ 0 & 9 & -10 & -49 \end{array}\right]$$ **Rewrote …**

$2*R_2 + R_1 \to R_1$

$-9R_2 + R_3 \to R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 4 \\ 0 & 1 & -1 & -5 \\ 0 & 0 & -1 & -4 \end{array}\right]$$

$-1*R_3 \to R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 4 \\ 0 & 1 & -1 & -5 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

$-1*R_3 + R_1 \to R_1$

$1*R_3 + R_2 \to R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

Solution: $(x, y, z) = (0, -1, 4)$

e. Determinants – See other notes

f. Matrix Inverse – see other notes

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OFFENDING COMMAND:

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