Finite Two-sided (& One-sided) Limits
Part II

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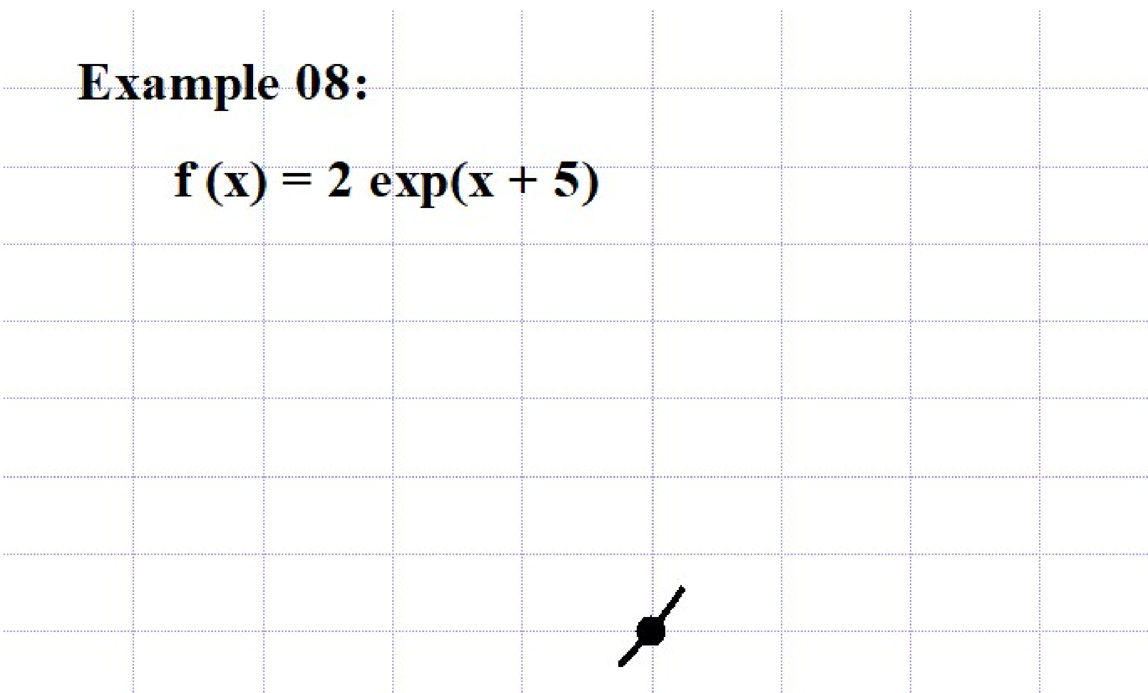
More good news: The limits of exponential, logarithmic, and their related functions as **x** approaches values in their domains is just the value of these functions at these domain values:

Example 08: Find $\lim\limits_{x \to -5} 2e^{x+5} \stackrel{?}{=} L$

Solution: We have

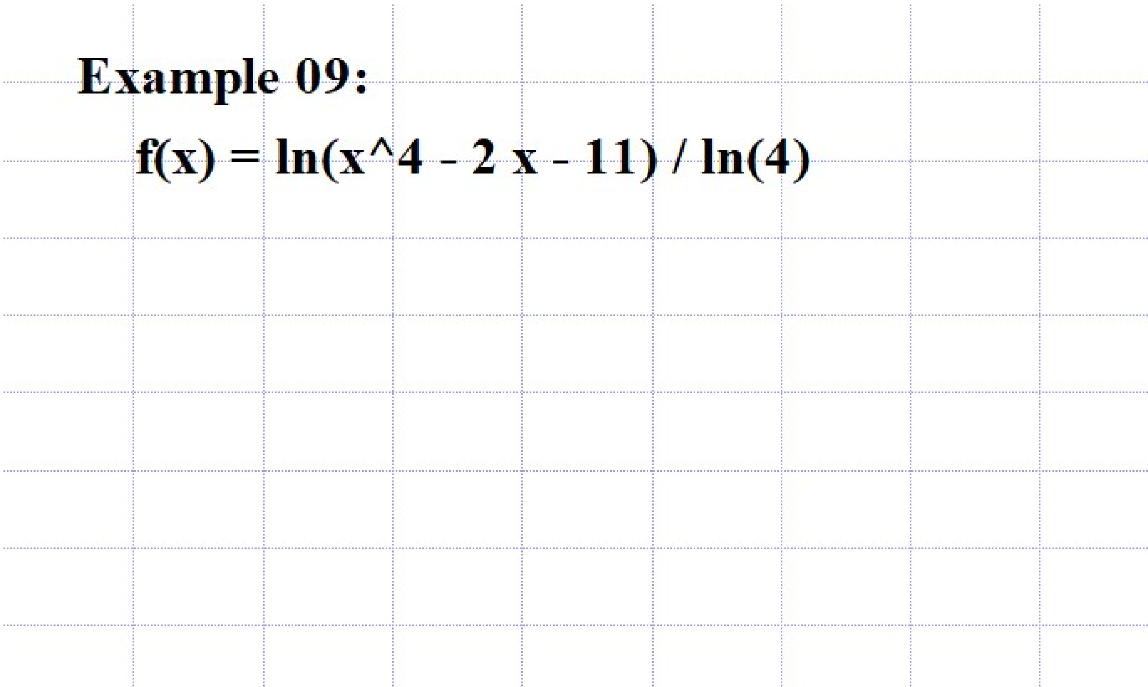
$$L = \lim_{x \to -5} 2e^{x+5} = 2e^{-5+5} = 2e^{0} = 2$$



Example 09: Find $\lim\limits_{\mathbf{x}\to 3} \log_4\left(\mathbf{x}^4 - 2\mathbf{x} - 11\right) \stackrel{?}{=} \mathbf{L}$

Solution: We have

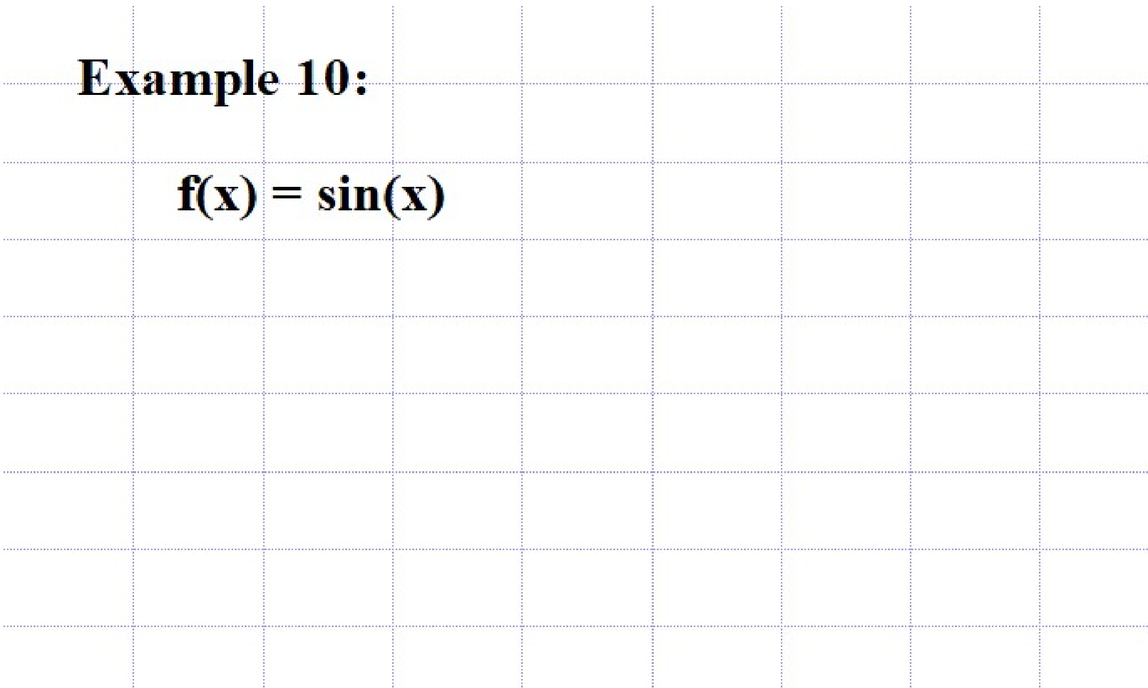
$$\mathbf{L} = \lim_{\mathbf{x}\to 3} \log_4\left(\mathbf{x}^4 - 2\mathbf{x} - 11\right) = \log_4 64 = \log_4 4^3 = 3\log_4 4 = 3$$



Also good news: The limits of the six trigonometric functions and their related functions as **x** approaches values in their domains is just the value of these functions at these domain values:

Example 10: Find ????
Solution: We have

$$\mathbf{L} = \mathop{\mathbf{Lim}}_{\mathbf{x} \to \frac{\pi}{2}} \sin \mathbf{x} = 1$$

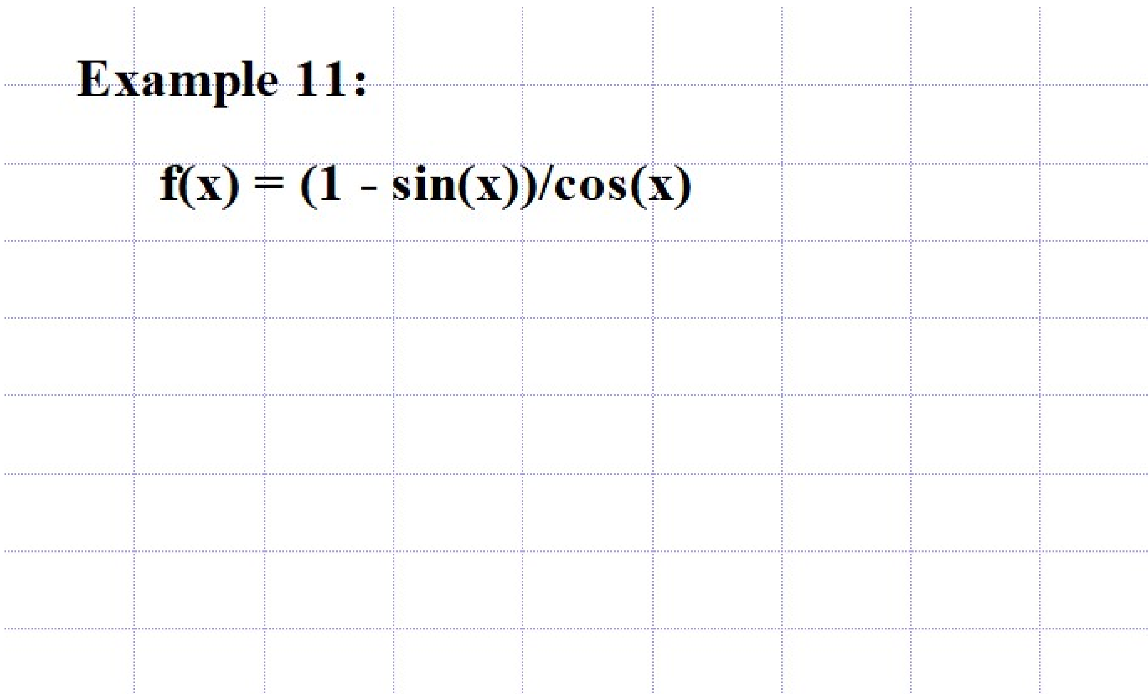


Example 11: Find $\lim_{x\to 0} \frac{1-\sin x}{\cos x} \stackrel{?}{=} L$

Solution: Since

$$\lim_{x\to 0}(1-\sin x) = 1 \;\&\; \lim_{x\to 0}\cos x = 1$$

the Quotient Theorem yields $L = \lim_{x\to 0} \frac{1-\sin x}{\cos x} = 1$



Within Calculus I (& Calculus II especially), the following trigonometric Identities are required:

FUNdamental Trigonometric Identity: $\sin^2 \mathbf{x} + \cos^2 \mathbf{x} = 1$

FUNdamental Inverse Trigonometric Identity: $\arcsin \mathbf{x} + \arccos \mathbf{x} = \dfrac{\pi}{2}$

In addition, the following limit, presented without proof, is fundamental in evaluating certain limits:

FUNdamental Trigonometric Limit:

a. $\mathbf{Lim}_{\mathbf{x}\to 0} \dfrac{\sin \mathbf{x}}{\mathbf{x}} = 1$; $\mathbf{Lim}_{\mathbf{x}\to 0} \dfrac{\mathbf{x}}{\sin \mathbf{x}} = 1$ **{Standard Form}**

b. $\mathbf{Lim}_{\mathbf{u}\to 0} \dfrac{\sin \mathbf{u}}{\mathbf{u}} = 1$; $\mathbf{Lim}_{\mathbf{u}\to 0} \dfrac{\mathbf{u}}{\sin \mathbf{u}} = 1$ **{Substitution Form}**

Example 12: Find $\lim\limits_{x\to 0} \dfrac{\sin 3x}{\sin 4x} \overset{?}{=} L$

Solution: Note the two (2) applications of the **Substitution Form**:

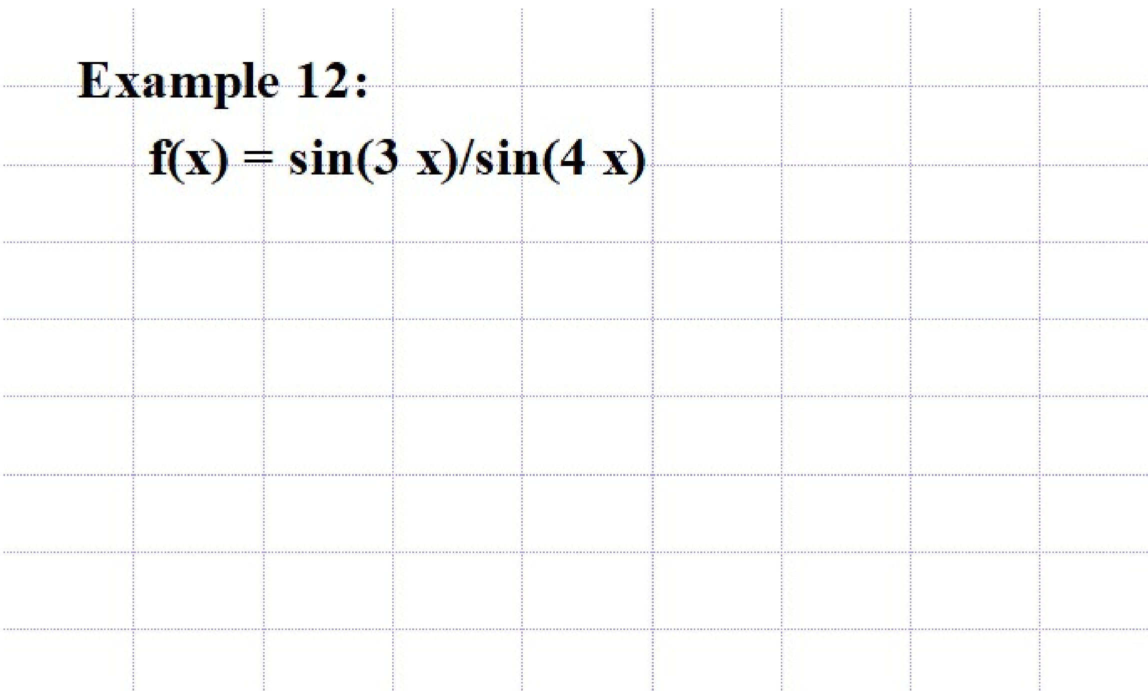
$$L = \lim_{x\to 0}\frac{\sin 3x}{\sin 4x} = \lim_{x\to 0}\frac{\sin 3x}{3x}\frac{4x}{\sin 4x}\frac{3x}{4x}$$

$$= \lim_{3x\to 0}\frac{\sin 3x}{3x} * \lim_{4x\to 0}\frac{4x}{\sin 4x} * \lim_{x\to 0}\frac{3x}{4x}$$

$$= 1 * 1 * \lim_{x\to 0}\frac{3}{4}$$

$$= \frac{3}{4} \quad \textbf{Converges "C"}$$

We used both the Product and Constant Theorems in our evaluation.

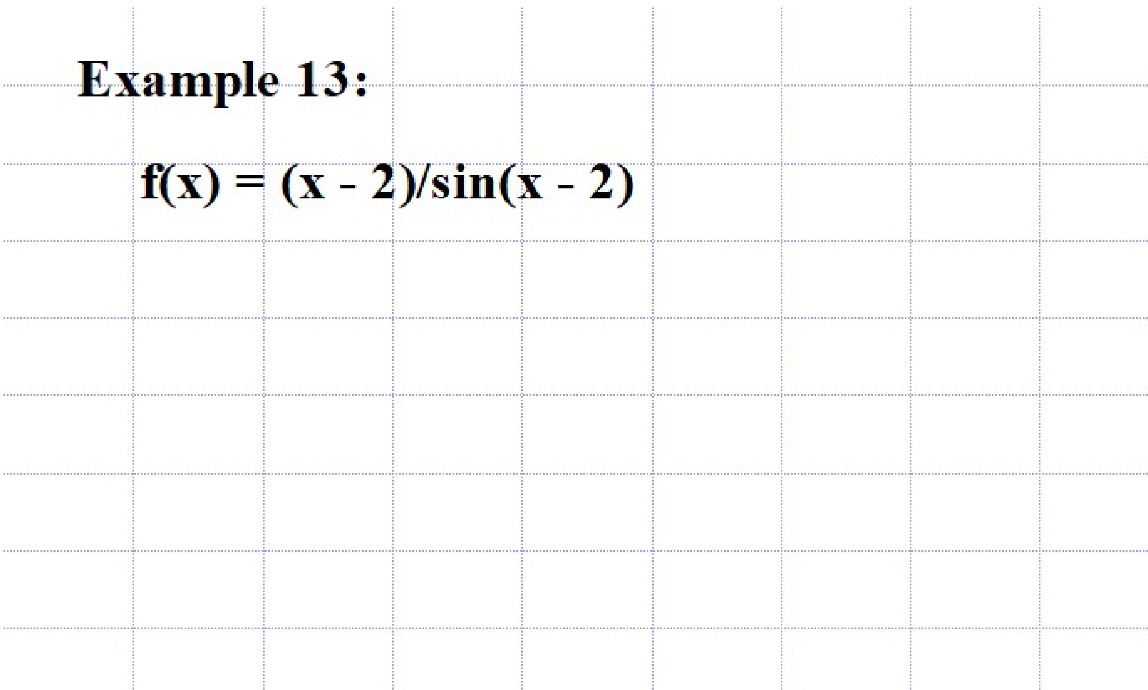


Example 13: Find $\underset{x\to 2}{\textbf{Lim}}\ \dfrac{\mathbf{x}-2}{\sin(\mathbf{x}-2)} \overset{?}{=} \mathbf{L}$

Solution: We use the **Substitution Form:**

$$\text{Set } \mathbf{u}=\mathbf{x}-2 \text{ so that } \mathbf{x}\to 2 \Rightarrow \mathbf{x}-2\to 0 \Rightarrow \mathbf{u}\to 0$$

$$\Rightarrow \mathbf{L} = \underset{\mathbf{u}\to 0}{\textbf{Lim}}\ \frac{\mathbf{u}}{\sin \mathbf{u}} = 1 \quad \textbf{Converges "C"}$$



Example 14: Find $\lim\limits_{x \to 3} \dfrac{\sin(x-3)}{x^2 - 5x + 6} \stackrel{?}{=} L$

Solution: We use factoring and the **Substitution Form:**

Since $x^2 - 5x + 6 = (x-3)\,(x-2)$, we set

$$u = x - 3 \Rightarrow x = u + 3 \Rightarrow x - 2 = u + 1$$

Thus $x \to 3 \Rightarrow x - 3 \to 0 \Rightarrow u \to 0$

$$L = \lim_{x \to 3} \frac{\sin(x-3)}{x^2 - 5x + 6} = \lim_{x \to 3} \frac{\sin(x-3)}{(x-3)\,(x-2)}$$

$$= \lim_{u \to 0} \frac{\sin u}{u\,(u+1)}$$

$$= \lim_{u \to 0} \frac{\sin u}{u} * \lim_{u \to 0} \frac{1}{u+1}$$

$$= 1 \quad \textbf{Converges "C"}$$