Equations – Introduction

Left-hand Side = Right-hand Side

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An **equation** is a statement that the expression on the left-hand side equals ("=") the expression on the right-hand side. The following are a few of the many types of equations:

1. $3x-4=11$; linear equation ["x" is to the power "1": $x^1=x$]
2. $3-(x+4)=2(3-x)$; linear equation
3. $x^2-2x=8$; quadratic equation ["x" is to the power "2": x^2]
4. $|2x-7|=3$; absolute value equation: $|?|$
5. $\sqrt{5x+6}=4$; radical equation: $\sqrt{?}$; in general, $\sqrt[n]{?}$
6. $\dfrac{x}{x-2}=2$; rational equation: $\dfrac{?}{?}$
7. $2^{x-3}=8$; exponential equation: $\text{Base}^{\text{Power}}$
8. $\log_2(x+25)=7$; logarithmic equation: $\log_{\text{Base}}(?)$

Notice that each of these equations has the letter "x" in them (actually, other letters may be used); it is called the **unknown**. The *goal* is the **solve** the equation. **Solve** means to find one or more values for "x", if there are any, that make the expression on the left-hand side equal ("=") to the expression on the right-hand side. These values will be called **solutions** of the equation. We are NOT ready to learn how to solve all of the equations above yet, but we can determine if a particular value for "x" is a solution.

1. Show that x = 5 is a solution of the linear equation $3x-4=11$. We just substitute x = 5 into the left-hand side of the equation and see if we get 11.
   Let's see
   $$3x-4=3*5-4=15-4=11$$
   so x = 5 is a solution.

2. Is x = 7 a solution of $3-(x+4)=2(3-x)$? Substituting x = 7 in both sides of the equation yields

$$3-(7+4)\overset{?}{=}2(3-7)$$

$$3-11\overset{?}{=}2(-4)$$

$$-8\overset{\text{YES}}{=}-8$$

so x = 7 is a solution.

3. Which of the following are solutions of the quadratic equation $x^2-2x=8$:
   a. x = -2
   b. x = 2
   c. x = 4

We have
   a. x = - 2: YES!
      $(-2)^2-2*(-2)=4+4=8$
   b. x = 2: NO!
      $(2)^2-2*(2)=4-4=0$
   c. x = 4: YES!
      $(4)^2-2*(4)=16-8=8$
      [FYI: quadratic equations have two (2) solutions]

4. Which of the following are solutions of the absolute value equation $|2x-7|=3$:
   a. x = -1
   b. x = 2
   c. x = 5

We have
   a. x = - 1: NO!
      $|2*(-1)-7|=|-2-7|=|-9|=9\neq 3$
   b. x = 2: YES!

$$|2*2-7|=|4-7|=|-3|=3$$

c. x = 5: YES!

$$|2*5-7|=|10-7|=|3|=3$$

5. Which of the following are solutions of the radical equation $\sqrt{5x+6}=4$:
   a. x = -2
   b. x = 0
   c. x = 2

   We have
   a. x = -2: NO!
      $$\sqrt{5*(-2)+6}=\sqrt{-10+6}=\sqrt{-10}\neq 4 \;;$$
      [No negatives under square root until College Algebra – complex numbers]
   b. x = 0: NO!
      $$\sqrt{5*0+6}=\sqrt{6}\approx 2.45\neq 4 \;;$$
      [Calculator used to get approximation: $\sqrt{6}\approx 2.45$]
   c. x = 2: YES!
      $$\sqrt{5*2+6}=\sqrt{10+6}=\sqrt{16}=4$$
      [We *always* take the positive square root: $\sqrt{16}=4$]

6. Which of the following are solutions of the rational equation $\frac{x}{x-2}=2$:
   a. x = 0
   b. x = 2
   c. x = 4

   We have
   a. x = 0: NO!
      $$\frac{0}{0-2}=\frac{0}{-2}=0\neq 2 \;;$$
   b. x = 2: NO!

$$\frac{2}{2-2} = \frac{2}{0} = \text{Undefined} \neq 2 \;;$$

c. x = 4: YES!

$$\frac{4}{4-2} = \frac{4}{2} = 2$$

[Note: We can NOT divide out the "4": $\frac{\not{4}}{\not{4}-2}$]

7. Which of the following are solutions of the exponential equation:
   a. x = 0
   b. x = 3
   c. x = 6

We have

a. x = 0: NO!

$$2^{0-3} = 2^{-3} = \frac{1}{8} \neq 8 \;;$$

b. x = 3: NO!

$$2^{3-3} = 2^{0} = 1 \neq 8 \;;$$

c. x = 6: YES!

$$2^{6-3} = 2^{3} = 8$$

8. We will consider logarithmic equations later.

YES, you can probably guess the solutions to some of these equations. But PLEASE concentrate on using the properties of arithmetic/algebra to verify all the calculations as you will NOT be able to guess the solutions of the harder equations we will study later.