Equations – Quadratic: Introduction

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Goal: Solve Quadratic Equations. This is the main goal but first we will discuss the nature of their solutions and give procedures for solving a couple of trivial quadratic equations.

Standard Form:

$$ax^2 + bx + c = 0$$

where $a \neq 0, b, c$ are real numbers. Sometimes the quadratic equation does NOT come in standard form but we rewrite it in this form before starting the solution procedure.

Discriminate: $D = b^2 - 4ac$ gives us information about the solutions:

Quadratic Equations have two (2) solutions:

1. **Real Numbers:**
   a. Same real number twice: $D = b^2 - 4ac = 0$
   b. Different real numbers: $D = b^2 - 4ac > 0$
2. **Complex Numbers** – must occur in *conjugate* pairs:
   $D = b^2 - 4ac < 0$

Depending upon the values of a,b,c there are different techniques to solve the equation:

1. If $b = 0$: **Square Root Technique**

b = 0	a = 4 ; b = 0 ; c = -9
$ax^2 + c = 0$	$4x^2 - 9 = 0$
$x^2 = -\frac{c}{a}$	$x^2 = \frac{9}{4}$
$x = \pm\sqrt{-\frac{c}{a}}$	$x = \pm\sqrt{\frac{9}{4}} = \pm\frac{3}{2}$ (2 solutions)

2. If $c=0$: **Factor Technique**

c = 0	a = 5 ; b = 3 ; c = 0
$ax^2+bx=0$	$5x^2+3x=0$
$x(ax+b)=0$	$x(5x+3)=0$
$x=0 \;\mid\; x=-\frac{b}{a}$	$x=0 \;\mid\; x=-\frac{3}{5}$

3. If $a\neq 0\,;\,b\neq 0\,;\,c\neq 0$: **Three Techniques**
   Many more details about these techniques forthcoming.

   a. **Factor** and set each factor equal to zero

   Zero Product Help –

   $$\square * \triangle = 0 \;\Rightarrow\; \square = 0 \text{ or } \triangle = 0 \text{ or both equal } 0$$

   Example:

   $x^2+2x-15=0 \Rightarrow (x-3)(x+5)=0$
   $\Rightarrow x-3=0\,;\,x+5=0$
   $\Rightarrow x=-5,3$

   Factor Help –

   a. $(x+b)(x+d)=x^2+(b+d)x+bd$
   Example:

   $x^2+2x-15=0 \Rightarrow bd=-15$ & $b+d$ MUST be 2

   Build table …

b	d	b + d
15	-1	14
-15	1	-14
3	-5	-2
-3	**5**	**2 (YES!)**

$\therefore\; x^2+2x-15=0 \Rightarrow (x-3)(x+5)=0$

b. $(ax+b)(cx+d) = ac\,x^2 + (bc+ad)\,x + bd$

Example:

$8x^2 + 2x - 15 = 0 \Rightarrow ac = 8, bd = -15$ & $bc + ad$ MUST be 2

Build table:

a	c	b	d	bc + ad
2	4	-15	1	-58
2	4	15	-1	58
2	4	-3	5	-2
2	**4**	**3**	**-5**	**2 (YES!)**

$\therefore 8x^2 + 2x - 15 = 0 \Rightarrow (2x+3)(4x-5) = 0$

b. **Complete the Square** & then use the Square Root Technique

Perfect Square:

$(x+a)^2 = x^2 + 2a\,x + a^2$

$= 1 * x^2 + 2a\,x + a^2$

$= [x^2\text{- coefficient}] * x^2 + [x\text{- coefficient}] * x + [\text{constant}]$

$\therefore x^2\text{- coefficient} = 1$; $x\text{- coefficient} = 2a$; $\text{constant} = a^2$

Important:

1. $x^2\text{- coefficient} = 1$

2. $x\text{- coefficient} = 2a$

$\frac{1}{2}\left(x\text{- coefficient}\right) = a$

$\left[\frac{1}{2}\left(x\text{- coefficient}\right)\right]^2 = a^2 = \text{constant}$ (last term)

Example:

Steps	Example
$x^2+2ax=\#$	$x^2+6x-7=0 \Rightarrow x^2+6x=7$
$x^2+2ax+\left[\frac{1}{2}(x-coeff)\right]^2$ $=\# \;+\left[\frac{1}{2}(x-coeff)\right]^2$	$x^2+6x+\left(\frac{6}{2}\right)^2=7+\left(\frac{6}{2}\right)^2$
$x^2+2ax+a^2=\# \;+\; a^2$	$x^2+6x+9=7+9$
$(x+a)^2=\# \;+\; a^2$	$(x+3)^2=16$
$x+a=\pm\sqrt{\# \;+\; a^2}$	$x+3=\pm\sqrt{16}=\pm 4$ (2 Solutions!)
$x=-a\pm\sqrt{\# \;+\; a^2}$	$x=-3\pm 4=\begin{cases}-7\\1\end{cases}$

c. Use the **Quadratic Formula** (**QF**): $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Arithmetic will try to lower your grade & mine!

Example:

$x^2 + 6x - 7 = 0 \Rightarrow a = 1\ ;\ b = 1\ ;\ c = -7$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-6 \pm \sqrt{6^2 - 4(1)(-7)}}{2(1)} \\ &= \frac{-6 \pm \sqrt{36 + 28}}{2} \\ &= \frac{-6 \pm \sqrt{64}}{2} \\ &= \frac{-6 \pm 8}{2} \\ &= \begin{cases} \dfrac{-6 - 8}{2} = -7 \\ \dfrac{-6 + 8}{2} = 1 \end{cases} \end{aligned}$$

