Equations – Quadratic: Solve

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Goal: Solve various Quadratic Equations using various techniques.

(1) **Question**: Solve the equation $4\mathbf{x}^2 - 81 = 0$.
Solution:

Note: $\mathbf{a} = 4$; $\mathbf{b} = 0$; $\mathbf{c} = -81$

Step	Equation	Reason
0	$4\mathbf{x}^2 - 81 = 0$	
1	$4\mathbf{x}^2 = 81$	
2	$\mathbf{x}^2 = \frac{81}{4}$	
3	$\mathbf{x} = \pm\sqrt{\frac{81}{4}} = \pm\frac{9}{2}$	2 Solutions

Graph of solution set:

________ −9 / 2 ________ \| ________ 9 / 2 ____________
0

Note: there are certainly other ways to solve this and other equations.

(2) **Question**: The solution(s) of the equation $\frac{\mathbf{x}^2}{9}+\frac{1}{16}=0$ satisfies

A) One real solution ("root of multiplicity two")
B) Two real solutions ("distinct solutions")
C) Two complex solutions (NOT conjugate pairs)
D) Two complex solutions (Conjugate pairs)

Solution:

Using the discriminate $\mathbf{D}=\mathbf{b}^2-4\mathbf{ac}$ with $\mathbf{a}=\frac{1}{9}$; $\mathbf{b}=0$;$\mathbf{c}=\frac{1}{16}$, we have

$$\mathbf{D}=0-4\left(\frac{1}{9}\right)\left(\frac{1}{16}\right)<0$$

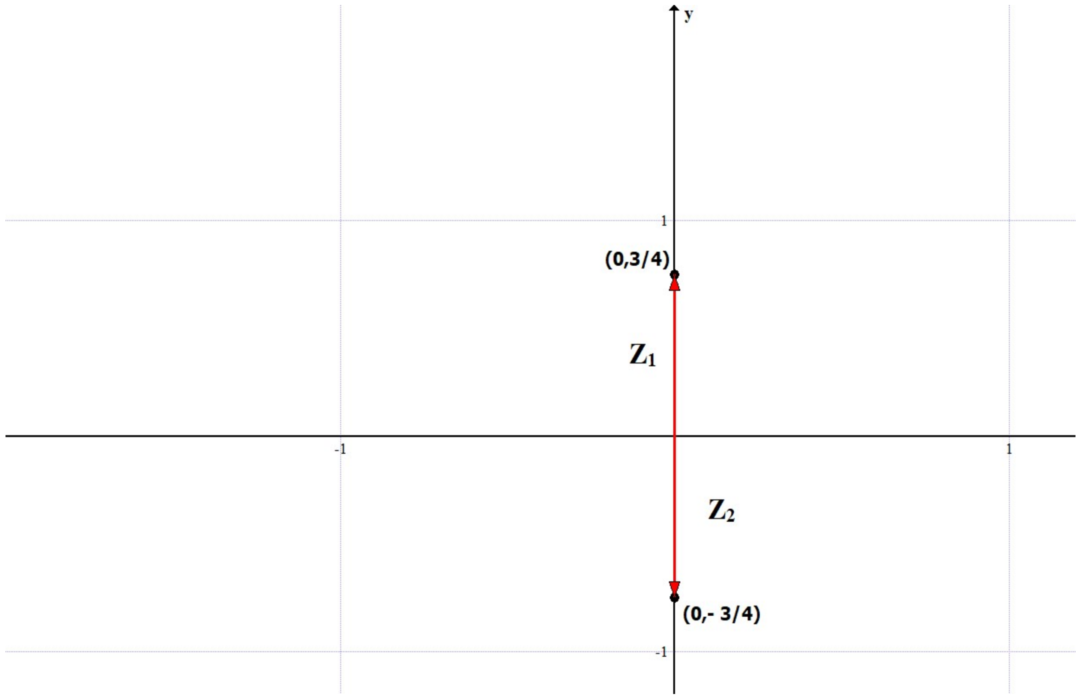
So that the answer is "D since complex solutions *must* occur in conjugate pairs

OR

we can just solve the equation and get the same result:

Step	Equation	Reason
0	$\frac{\mathbf{x}^2}{9}+\frac{1}{16}=0$	
1	$\frac{\mathbf{x}^2}{9}=-\frac{1}{16}$	
2	$\mathbf{x}^2=-\frac{9}{16}$	
3	$\mathbf{x}=\pm\sqrt{-\frac{9}{16}}$	
4	$\mathbf{x}=\pm\sqrt{\frac{9}{16}}*\sqrt{-1}$	
5	$\mathbf{x}=\pm\frac{3}{4}\mathbf{i}$	

Graph of the solution set:



(3) **Question**: Find the *smallest* solution of the equation $x^2 - 14x + 48 = 0$.
Solution:

Note: $a = 1$; $b = -14$; $c = 48$

Three (3) solutions are given below:

Factor:

Step	Equation	Reason
0	$x^2 - 14x + 48 = 0$	
1	$(x-6)(x-8) = 0$	
2	$x - 6 = 0$ ‖ $x - 8 = 0$ $x = 6$ ‖ $x = 8$	

Complete the Square:

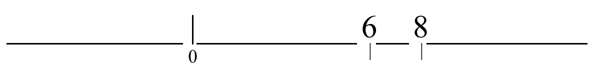
Step	Equation	Reason
0	$x^2 - 14x + 48 = 0$	
1	$x^2 - 14x = -48$	x-coeff: -14 1/2 (x-coeff): -7 $[1/2(\text{x-coeff})]^2$: 49
2	$x^2 - 14x + 49 = -48 + 49$	

3	$\left(\mathbf{x}-7\right)^2=1$	
4	$\mathbf{x}-7=\pm 1$	
5	$\mathbf{x}-7=-1$ ‖ $\mathbf{x}-7=1$ $\mathbf{x}=6$ ‖ $\mathbf{x}=8$	

Quadratic Formula - QF:

Step	Equation	Reason
0	$\mathbf{x}^2-14\mathbf{x}+48=0$	
1	$\mathbf{x}=\dfrac{-\mathbf{b}\pm\sqrt{\mathbf{b}^2-4\mathbf{ac}}}{2\mathbf{a}}$	
2	$\mathbf{x}=\dfrac{-\left[-14\right]\pm\sqrt{\left[-14\right]^2-4\left[1\right]\left[48\right]}}{2\left[1\right]}$	
3	$\mathbf{x}=\dfrac{14\pm\sqrt{196-192}}{2}$	
4	$\mathbf{x}=\dfrac{14\pm\sqrt{4}}{2}$	
5	$\mathbf{x}=\dfrac{14\pm 2}{2}$	
6	$\mathbf{x}=\dfrac{14-2}{2}$ ‖ $\mathbf{x}=\dfrac{14+2}{2}$ $\mathbf{x}=6$ ‖ $\mathbf{x}=8$	

Graph of the solution set:



Answer is x = 6.

(4) **Question**: Find the *sum* of the solutions of the equation $10\mathbf{x}^2-11\mathbf{x}=6$.
Solution:

Standard Form: $10\mathbf{x}^2-11\mathbf{x}-6=0$ so that $\mathbf{a}=10$; $\mathbf{b}=-11$; $\mathbf{c}=-6$

Again, we find three (s) solutions:

Factor:

Step	Equation	Reason
0	$10\mathbf{x}^2 - 11\mathbf{x} - 6 = 0$	
1	$(5\mathbf{x}+2)(2\mathbf{x}-3) = 0$	
2	$5\mathbf{x}+2=0$, $5\mathbf{x}=-2$, $\mathbf{x}=-\frac{2}{5}$ ‖ $2\mathbf{x}-3=0$, $2\mathbf{x}=3$, $\mathbf{x}=\frac{3}{2}$	

Complete the Square:

Step	Equation	Reason
0	$10\mathbf{x}^2 - 11\mathbf{x} - 6 = 0$	
1	$10\left[\mathbf{x}^2 - \frac{11}{10}\mathbf{x}\right] = 6$	x-coeff: $-\frac{11}{10}$ 1/2 (x-coeff): $-\frac{11}{20}$ $[1/2(\text{x-coeff})]^2$: $\frac{121}{400}$
2	$10\left[\mathbf{x}^2 - \frac{11}{10}\mathbf{x} + \frac{121}{400}\right] = 6 + 10 * \frac{121}{400}$	
3	$10\left[\mathbf{x} - \frac{11}{20}\right]^2 = \frac{361}{40}$	
4	$\left[\mathbf{x} - \frac{11}{20}\right]^2 = \frac{361}{400}$	
5	$\mathbf{x} - \frac{11}{20} = \pm\sqrt{\frac{361}{400}} = \pm\frac{19}{20}$	
6	$\mathbf{x} - \frac{11}{20} = -\frac{19}{20}$, $\mathbf{x} = \frac{11}{20} - \frac{19}{20}$, $\mathbf{x} = -\frac{8}{20} = -\frac{2}{5}$ ‖ $\mathbf{x} - \frac{11}{20} = \frac{19}{20}$, $\mathbf{x} = \frac{11}{20} + \frac{19}{20}$, $\mathbf{x} = \frac{30}{20} = \frac{3}{2}$	

Quadratic Formula - QF:

Step	Equation	Reason
0	$10\mathbf{x}^2 - 11\mathbf{x} - 6 = 0$	
1	$\mathbf{x} = \dfrac{-\mathbf{b} \pm \sqrt{\mathbf{b}^2 - 4\mathbf{ac}}}{2\mathbf{a}}$	
2	$\mathbf{x} = \dfrac{-[-11] \pm \sqrt{[-11]^2 - 4[10][-6]}}{2[10]}$	
3	$\mathbf{x} = \dfrac{11 \pm \sqrt{121 + 240}}{20}$	
4	$\mathbf{x} = \dfrac{11 \pm \sqrt{361}}{20}$	
5	$\mathbf{x} = \dfrac{11 \pm 19}{20}$	
6	$\mathbf{x} = \dfrac{11 - 19}{20}$ ‖ $\mathbf{x} = \dfrac{11 + 19}{20}$ $\mathbf{x} = -\dfrac{5}{2}$ ‖ $\mathbf{x} = \dfrac{3}{2}$	

Graph of the solution set:

________ $-2/5$ ___ | ____________________ $3/2$ ____________
0

The **sum** is $\text{Sum} = -\frac{2}{5} + \frac{3}{2} = -\frac{2*2}{5*2} + \frac{3*5}{2*5} = -\frac{4}{10} + \frac{15}{10} = \frac{11}{10}$

(5) **Question**: Find the *largest* solution $\mathbf{x}_{\text{Large}}$ of the equation $3\mathbf{x}^2 + 2\mathbf{x} - 4 = 0$.
Solution:

Note: $\mathbf{a} = 3$; $\mathbf{b} = 2$; $\mathbf{c} = -4$

Only two (2) solutions are given:

Factor:

Doesn't factor "nicely"! **UGLY!**

Complete the Square:

Step	Equation	Reason
0	$3\mathbf{x}^2 + 2\mathbf{x} - 4 = 0$	
1	$3\left[\mathbf{x}^2 + \frac{2}{3}\mathbf{x}\right] = 4$	x-coeff: $\frac{2}{3}$ 1/2 (x-coeff): $\frac{1}{3}$ [1/2(x-coeff)]^2: $\frac{1}{9}$
2	$3\left[\mathbf{x}^2 + \frac{2}{3}\mathbf{x} + \frac{1}{9}\right] = 4 + 3 * \frac{1}{9}$	
3	$3\left[\mathbf{x} + \frac{1}{3}\right]^2 = \frac{13}{3}$	
4	$\left[\mathbf{x} + \frac{1}{3}\right]^2 = \frac{13}{9}$	
5	$\mathbf{x} + \frac{1}{3} = \pm\sqrt{\frac{13}{9}} = \pm\frac{\sqrt{13}}{3}$	
6	$\mathbf{x} + \frac{1}{3} = -\frac{\sqrt{13}}{3}$ $\mathbf{x} = -\frac{1+\sqrt{13}}{3}$ $\mathbf{x} \approx -1.5352$ ‖ $\mathbf{x} + \frac{1}{3} = \frac{\sqrt{13}}{3}$ $\mathbf{x} = \frac{-1+\sqrt{13}}{3}$ $\mathbf{x} \approx 0.8685$	

Quadratic Formula - QF:

Step	Equation	Reason
0	$3\mathbf{x}^2 + 2\mathbf{x} - 4 = 0$	
1	$\mathbf{x} = \frac{-\mathbf{b} \pm \sqrt{\mathbf{b}^2 - 4\mathbf{ac}}}{2\mathbf{a}}$	
2	$\mathbf{x} = \frac{-[2] \pm \sqrt{[-2]^2 - 4[3][-4]}}{2[3]}$	
3	$\mathbf{x} = \frac{-2 \pm \sqrt{4+48}}{6}$	

4	$\mathbf{x} = \dfrac{-2 \pm \sqrt{52}}{6} = \dfrac{-2 \pm 2\sqrt{13}}{6}$	
5	$\mathbf{x} = \dfrac{-1 \pm \sqrt{13}}{3}$	
6	$\mathbf{x} = \dfrac{-1 - \sqrt{13}}{3}$ $\mathbf{x} \approx -1.5352$ ‖ $\mathbf{x} = \dfrac{-1 + \sqrt{13}}{3}$ $\mathbf{x} \approx 0.8685$	

Graph of the solution set:

________ −1.54 __________________ |0 _________ 0.87 ____________

Therefore $\mathbf{x}_{\mathbf{Large}} = \dfrac{-1 + \sqrt{13}}{3}$.

(6) **Question**: Is there a *largest* solution $\mathbf{x}_{\mathbf{Large}}$ of the equation $\mathbf{x}^2 + 3\mathbf{x} + 5 = 0$?
Solution:

Note: $\mathbf{a} = 1$; $\mathbf{b} = 3$; $\mathbf{c} = 5$

Ugly again!

Factor:

Doesn't factor "nicely"! **UGLY!**

Complete the Square:

Step	Equation	Reason
0	$\mathbf{x}^2+3\mathbf{x}+5=0$	
1	$\mathbf{x}^2+3\mathbf{x}=-5$	x-coeff: 3 1/2 (x-coeff): $\frac{3}{2}$ [1/2(x-coeff)]^2: $\frac{9}{4}$
2	$\mathbf{x}^2+3\mathbf{x}+\frac{9}{4}=-5+\frac{9}{4}$	
3	$\left(\mathbf{x}+\frac{3}{2}\right)^2=-\frac{11}{4}$	
4	$\mathbf{x}+\frac{3}{2}=\pm\sqrt{-\frac{11}{4}}$	
5	$\mathbf{x}+\frac{3}{2}=\pm\frac{\sqrt{-11}}{2}=\pm\frac{\sqrt{11}\,\mathbf{i}}{2}$	
6	$\mathbf{x}+\frac{3}{2}=-\frac{\sqrt{11}\,\mathbf{i}}{2}$ $\quad\Vert\quad$ $\mathbf{x}+\frac{3}{2}=\frac{\sqrt{11}\,\mathbf{i}}{2}$ $\mathbf{x}=-\frac{3+\sqrt{11}\,\mathbf{i}}{2}$ $\quad\Vert\quad$ $\mathbf{x}=\frac{-3+\sqrt{11}\,\mathbf{i}}{2}$	

Quadratic Formula - QF:

Step	Equation	Reason
0	$\mathbf{x}^2+3\mathbf{x}+5=0$	
1	$\mathbf{x}=\frac{-\mathbf{b}\pm\sqrt{\mathbf{b}^2-4\mathbf{ac}}}{2\mathbf{a}}$	
2	$\mathbf{x}=\frac{-[3]\pm\sqrt{[3]^2-4[1][5]}}{2[1]}$	
3	$\mathbf{x}=\frac{-3\pm\sqrt{9-20}}{2}$	
4	$\mathbf{x}=\frac{-3\pm\sqrt{-11}}{2}$	
5	$\mathbf{x}=\frac{-3\pm\sqrt{11}\,\mathbf{i}}{2}$	

6	$\mathbf{x} = \frac{-3-\sqrt{11}\ \mathbf{i}}{2}$	$\mathbf{x} = \frac{-3+\sqrt{11}\ \mathbf{i}}{2}$	

There is no "largest" solution since the solutions are complex numbers.

Graph of the solution set: