Inequalities – Introduction

Equality (Equivalence) & Inequality (Non-Equality)

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1. Equality

The word equality comes from the word **equal** which states that two (2) items are the same. We use the sign = to denote equality (& equivalence):

Right Hand Side = Left and Side [RHS = LHS]

Denote **equality**

$$3 = 3$$

or

equivalence

$$3 = \frac{3}{1}$$

$$3 = \frac{12}{4}$$

$$3 = \sqrt{9}$$

$$3 = \sqrt[3]{27}$$

$$3 = |-3|$$

$$3 = 3.00$$

.

.

.

Note: With an equivalence, the values are the same but the forms are different. Different forms are used for different purposes.

Frequently in Mathematics, we are asked to **solve** an equality (called an **equation**) for a letter, say "x", which means that we are looking for a number or numbers, that make(s) the equation true:

$$2(\mathbf{x}-4)=13-\mathbf{x}$$

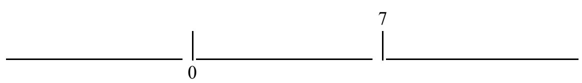
$$2([?]-4)=13-[?]$$

The solution is $\mathbf{x}=7$ since

$$2([7]-4)=13-[7]$$

$$6=6$$

Always draw the graph of the solution set:



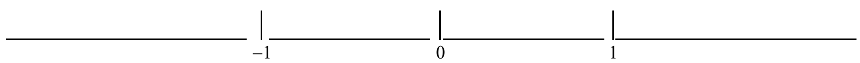
Note that sometimes in Mathematics, we must change the *form* but not the *value* in order "to do the math":

$$\begin{aligned}\frac{1}{6}+\frac{8}{3}&=\frac{1}{6}+\frac{8}{3}*1\\&=\frac{1}{6}+\frac{8}{3}*\frac{2}{2}\\&=\frac{1}{6}+\frac{16}{6}\\&=\frac{17}{6}\end{aligned}$$

FYI: The above "form changer" comes from the identities $\frac{a}{a}=1\,;\,a\neq 0$ & $a=a*1$. Another "form changer" is $a=a+0$

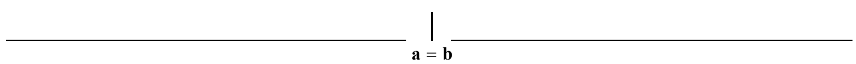
2. Inequality

If two (2) items are NOT equal or equivalent, we use the symbol $\neq$: $9 \neq 11$
Consider two (2) numbers "a" and "b" on the number line (think fancy "ruler"):



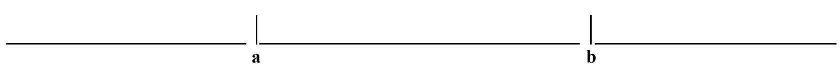
Number Line

If they are equal ($\mathbf{a} = \mathbf{b}$), they reside at the same location on the number line:



Number Line

If not, we write $\mathbf{a} \neq \mathbf{b}$ and one number must be to the left of the other one:

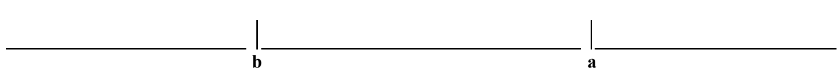


Number Line

In this case, we write

$a < b$ (or $b > a$)

and say that "a" is *less than* "b" (or "b" is *greater than* "a") If "b" is to the left of "a",



Number Line

we write $b < a$ (or $a > b$) and say that "a" is *greater than* "b" (or "b" is *less than* "a"). Hence,

$3 < 5$ (or $5 > 3$)
$7 > 4$ (or $4 < 7$)

Writing $x < 6$ means that "x" represents *any* number less than 6:

$$\overbrace{/\,/}^{\mathbf{x}})_{6}\underline{\qquad\qquad\qquad\qquad}$$

Note: ")" means excluded

For example

$$-62 < 6\ ;\ -\frac{3}{4} < 6\ ;\ 0 < 6\ ;\ 2\frac{1}{2} < 6$$

The statement 11 < 6 is FALSE which we write as $11 \not< 6$ **.**
Writing -2 < x means that “x” represents *any* number greater than -2:

$$\underline{\qquad\qquad\qquad\qquad}\ (_{-2}\overbrace{/\,/}^{\mathbf{x}}$$

For example

$$-2 < -1\frac{3}{5}\ ;\ -2 < 0\ ;\ -2 < 3\frac{2}{7}\ ;\ -2 < 73$$

Sometimes we want to allow equality so we write $\mathbf{a} \le \mathbf{b}$ (or $\mathbf{b} \ge \mathbf{a}$) which means a < b **OR** a = b:

$$\overbrace{/\,/\,/\,/\,/\,/\,/\,/\,/\,/\,/\,/\,/\,/\,/\,/\,/\,/\,/\,/}^{\mathbf{a}}]_{\mathbf{b}}\underline{\qquad\qquad\qquad\qquad}$$

Note: "]" means inclusion

We write $\mathbf{a} \ge \mathbf{b}$ (or $\mathbf{b} \le \mathbf{a}$) which means a > b **OR** a = b:

$$\underline{\qquad\qquad\qquad}\ [_{\mathbf{b}}\overbrace{/\,/\,/\,/\,/\,/\,/\,/\,/\,/\,/\,/\,/\,/\,/\,/\,/\,/\,/\,/}^{\mathbf{a}}$$

So

$$7 \le 13$$

since

$$7 < 13$$

and

$$7 \le 7$$

since

$$7 = 7$$

