Inequalities
Linear

MATH by Wilson
Your Personal Mathematics Trainer
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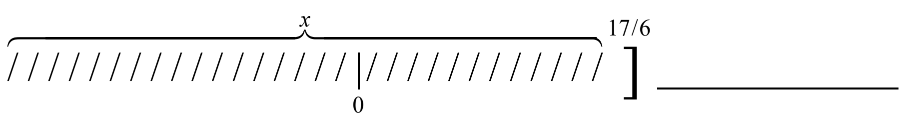
We consider two (2) types of linear inequalities:

1. One Inequality Symbol: $\{\leq ; < ; \geq ; >\}$

Question 01: Solve for x: $4-3(2+\mathbf{x}) \leq \mathbf{x}+5(3-2\mathbf{x})$
Solution:

Step	Inequality	Reason
0	$4-3(2+\mathbf{x}) \leq \mathbf{x}+5(3-2\mathbf{x})$	
1	$4-6-3\mathbf{x} \leq \mathbf{x}+15-10\mathbf{x}$	
2	$-2-3\mathbf{x} \leq -9\mathbf{x}+15$	
3	$9\mathbf{x}-3\mathbf{x} \leq 15+2$	
4	$6\mathbf{x} \leq 17$	
5	$\mathbf{x} \leq \frac{17}{6}$	

Graph of the solution set:



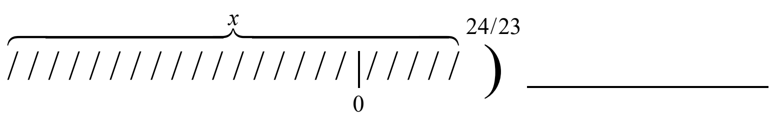
Question 02: Solve for x: $3-\frac{7}{3}\mathbf{x} > 4\left(\frac{3}{8}\mathbf{x}+1\right)-5$

Solution:

Step	Inequality	Reason
0	$3-\frac{7}{3}\mathbf{x} > 4\left(\frac{3}{8}\mathbf{x}+1\right)-5$	
1	$3-\frac{7}{3}\mathbf{x} > 4\left(\frac{3}{8}\mathbf{x}+1\right)-5=\frac{3\mathbf{x}}{2}-1$	

2	$\frac{3}{1}-\frac{7}{3}\mathbf{x}>\frac{3\mathbf{x}}{2}-\frac{1}{1}$	All Fractions
3	$6\left(\frac{3}{1}-\frac{7}{3}\mathbf{x}\right)>6\left(\frac{3\mathbf{x}}{2}-\frac{1}{1}\right)$	Multiply by Common Denominator " 6"
4	$18-14\mathbf{x}>9\mathbf{x}-6$	
5	$-14\mathbf{x}-9\mathbf{x}>-6-18$	
6	$-23\mathbf{x}>-24$	
7	$\mathbf{x}<\frac{-24}{-23}=\frac{24}{23}$	Note switch in direction

Graph of the solution set:



2. Two Inequality Symbols: $\{\le;<;\ge;>\}$ **- symbols pointing the same direction**

Question 03: Solve for x: $-4<1-3\mathbf{x}\le 4$
Solution:

Note: The solution of $-4<1-3\mathbf{x}\le 4$ **is actually the solution of two (2) inequalities**

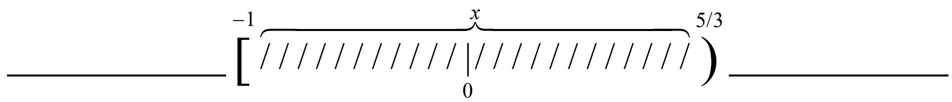
$-4<1-3\mathbf{x}$ and $1-3\mathbf{x}\le 4$

but they can be written and solved as $-4<1-3\mathbf{x}\le 4$ **since the inequality symbols are pointing in the same direction.**

Step	Inequality	Reason
0	$-4<1-3\mathbf{x}\le 4$	
1	$-4-1<-3\mathbf{x}\le 4-1$	
2	$-5<-3\mathbf{x}\le 3$	
3	$\frac{-5}{-3}>\mathbf{x}\ge\frac{3}{-3}$	**Divide by a negative Change direction of inequality symbol**

4	$\frac{5}{3} > \mathbf{x} \geq -1$ $-1 \leq \mathbf{x} < \frac{5}{3}$	

Graph of the solution set:



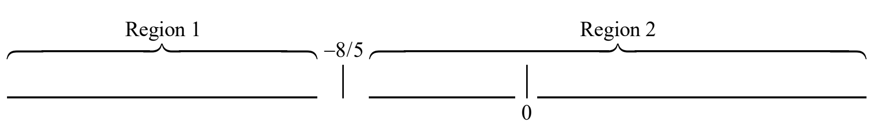
We now show an additional way to solve a linear equation when a solution exists. Consider

$$5-2(\mathbf{x}+3) \geq 3\mathbf{x}+7$$

First, we solve the corresponding linear equation

$$5-2(\mathbf{x}+3) = 3\mathbf{x}+7$$
$$5-2\mathbf{x}-6 = 3\mathbf{x}+7$$
$$-1-7 = 3\mathbf{x}+2\mathbf{x}$$
$$-8 = 5\mathbf{x}$$
$$\mathbf{x} = -\frac{8}{5}$$

Since the inequality is $"\geq"$**,** the number $\mathbf{x}=-\frac{8}{5}$ will be a solution. Now the number $\mathbf{x}=-\frac{8}{5}$, called a **boundary point**, divides the number line into two (2) regions:



One region, including $\mathbf{x}=-\frac{8}{5}$, will be the solution set; the other region will NOT. Now to find out, just pick a point in each region and determine which point satisfies the original inequality:

1. Choose, say x = -3:

$$5-2\left(\boxed{-3}+3\right)\overset{?}{\geq}3*\boxed{-3}$$

$$5\overset{?}{\geq}-9$$

TRUE!

2. Choose, say x = 0:

$$5-2\left(\boxed{0}+3\right)\overset{?}{\geq}3*\boxed{0}$$

$$-1\overset{?}{\geq}0$$

FALSE!

The solution set is $\left(-\infty,-\frac{8}{5}\right]$**:**

$$\overbrace{///////////////////////}^{x}\ \Big]^{-8/5}\ \_\_\_\_\_\_\_\_\_\_\_\_$$

