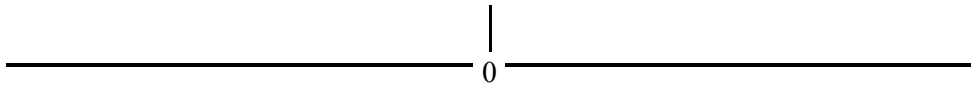


Inequalities Radical

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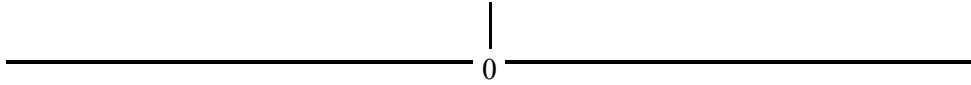
We are looking for the numbers on the horizontal number line (x-axis) that make the inequalities containing radicals ($\sqrt{?}$; $\sqrt[3]{?}$; ...) true. That is, when we are solving radical equations, we *only* consider solutions that are real numbers. We appeal to the following facts:

1. $u < 0 \Rightarrow \sqrt{u}$ is complex $u > 0 \Rightarrow \sqrt{u}$ is real & positive



$u = 0 \Rightarrow \sqrt{u} = 0$

2. $u < 0 \Rightarrow \sqrt[3]{u}$ is real & negative $u > 0 \Rightarrow \sqrt[3]{u}$ is real & positive



$u = 0 \Rightarrow \sqrt[3]{u} = 0$

3. Square Root : $\sqrt{u} \geq 0$; $\sqrt{u} \neq 0$

4. Cube Root: $\sqrt[3]{u} \leq 0$ or $\sqrt[3]{u} \geq 0$

Question 01: Solve for x: $\sqrt{3-4x} \geq 0$

Solution:

Step	Inequality	Reason
0	$\sqrt{3-4x} \geq 0$	$\Rightarrow 3x - 4 \geq 0$
1	$3 - 4x \geq 0$	
2	$3 \geq 4x$	

3	$\frac{3}{4} \geq x$ (or $x \leq \frac{3}{4}$)	
4	Solution: $\left(-\infty, \frac{3}{4}\right]$	

Graph of solution set:

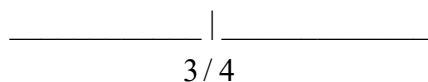


Question 02: Solve for x : $\sqrt{3-4x} \leq 0$

Solution:

Step	Inequality	Reason
0	$\sqrt{3-4x} \leq 0$	$\Rightarrow 3-4x = 0$
1	$3 = 4x$	
2	$\frac{3}{4} = x$ (or $x = \frac{3}{4}$)	
3	Solution: $\left\{\frac{3}{4}\right\}$	

Graph of solution set:



Question 03: Solve for x : $\sqrt[3]{3-4x} < 0$

Solution:

Step	Inequality	Reason
0	$\sqrt[3]{3-4x} < 0$	$\Rightarrow 3-4x < 0$
1	$3-4x < 0$	
2	$3 < 4x$	
3	$\frac{3}{4} < x$ (or $x > \frac{3}{4}$)	
4	Solution: $\left(\frac{3}{4}, +\infty\right)$	

Graph of solution set:



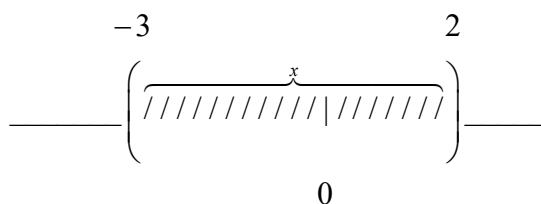
Question 04: Solve for x : $\sqrt{6-x-x^2} > 0$

Solution:

Step	Inequality	Reason
0	$\sqrt{6-x-x^2} > 0$	$\Rightarrow 6-x-x^2 > 0$
1	$6-x-x^2 > 0$	
2	$(3+x)(2-x) > 0$	
3	Determine Boundary Points: $(3+x)(2-x) = 0$ $x = -3 ; x = 2$	
4	Check Boundary Points: 1. $x = -3 : (3+[-3])(2-[-3]) > 0$ $(0)(5) > 0$ $0 > 0$ False ; -3 is NOT in the solution set 2. $x = 2 : (3+[2])(2-[2]) > 0$ $(5)(0) > 0$ $0 > 0$ False ; 2 is NOT in the solution set	

5	<p>Check Intervals:</p> <p>1. $(-\infty, -3)$: Test Point $x = -6$; $(3 + [-6])(2 - [-6]) > 0$ $(-3)(8) > 0$ $-24 > 0$ False; $(-\infty, -3)$ is NOT in the solution set</p> <p>2. $(-3, 2)$: Test Point $x = 0$; $(3 + [0])(2 - [0]) > 0$ $(3)(2) > 0$ $6 > 0$ True; $(-3, 2)$ is in the solution set</p> <p>3. $(2, +\infty)$: Test Point $x = 10$; $(3 + [10])(2 - [10]) > 0$ $(13)(-8) > 0$ $-104 > 0$ False; $(2, +\infty)$ is NOT in the solution set</p>	
6	Solution Set: $(-3, 2)$	

Graph of the solution set:



Question 05: Solve for x : $\sqrt[3]{\frac{x+4}{x}} \leq 4$

Solution:

Step	Inequality	Reason
0	$\sqrt[3]{\frac{x+4}{x}} \leq 4$	
1	$\left(\sqrt[3]{\frac{x+4}{x}}\right)^3 \leq 4^3$	
2	$\frac{x+4}{x} \leq 64$	
3	$\frac{x+4}{x} - \frac{64}{1} \leq 0$	

4	$\frac{x+4}{x} - \frac{64x}{x} \leq 0$	
5	$\frac{4-63x}{x} \leq 0$	
6	Boundary Points: 1) $4 - 63x = 0 \Rightarrow x = \frac{4}{63}$ in the solution set 2) $x = 0$ NEVER in the solution set	
7	Check Intervals: 1. $(-\infty, 0)$: Test Point $x = -10$; $\sqrt[3]{\frac{ -10 +4}{-10}} \stackrel{?}{\leq} 4$ $\sqrt[3]{\frac{-6}{-10}} = \sqrt[3]{\frac{3}{5}} \stackrel{?}{\leq} 4$ True 2. $(0, \frac{4}{63})$: Test Point $x = \frac{1}{63}$; $\sqrt[3]{\frac{ 1/63 +4}{1/63}} \stackrel{?}{\leq} 4$ $\sqrt[3]{253} \stackrel{?}{\leq} 4$ False 3. $(\frac{4}{63}, +\infty)$: Test Point $x = 10$; $\sqrt[3]{\frac{10+4}{10}} \stackrel{?}{\leq} 4$ $\sqrt[3]{\frac{10+4}{10}} = \sqrt[3]{\frac{7}{5}} \stackrel{?}{\leq} 4$ True	
8	Solution: $(-\infty, 0) \cup \left[\frac{4}{63}, +\infty\right)$	

Graph of the solution set:

