Rectangular Coordinate System with Distance & Midpoint Formulas

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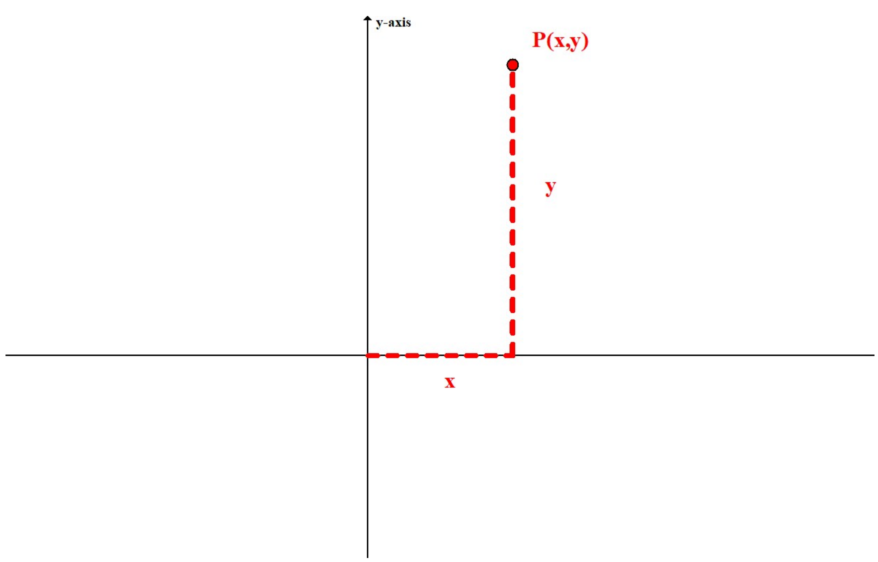
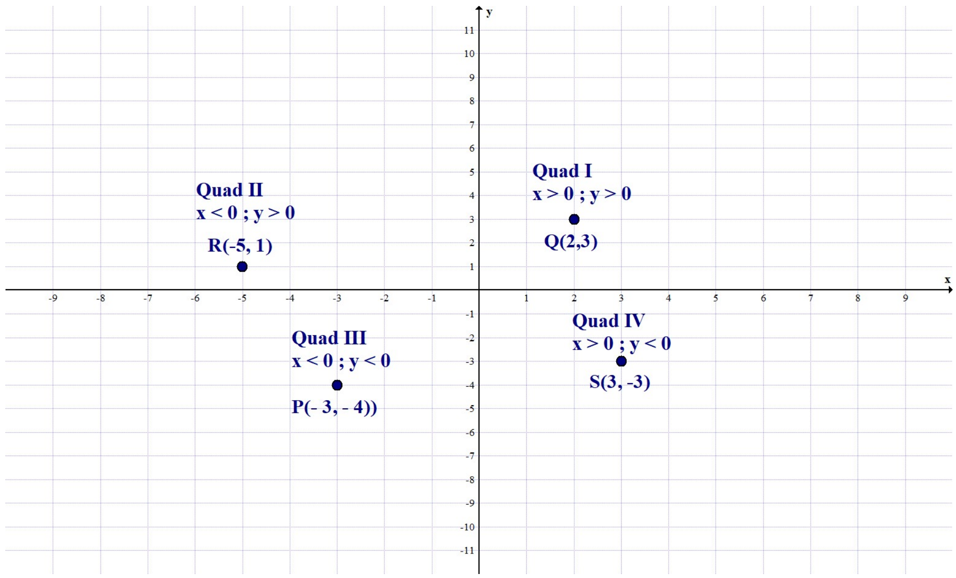
We have been considering the *horizontal* number line (called the **x-axis**) and have used it to graph the solution sets of various equations and inequalities. It is a (1) dimension graphical representation of the solution set.

If we now make a copy of it and rotate it 90 degrees counterclockwise, we get what we call the **y-axis**. We can now graph in two (2) dimensions. **Ordered pairs (points**) P(x, y) the form (x-coordinate , y-coordinate) are used to graph in what we call the **number plane** or in the **Cartesian Coordinate System**:



The x-axis and the y-axis divide the number plane into four (4) regions called **quadrants**:



Two (2) important formulas used throughout Mathematics are given below with an example:

Given points $\mathbf{P(x_1, y_1); Q(x_2, y_2)}$

1. The **distance between P and Q** is given by

   Distance between P & Q: d_{PQ}

   $$\mathbf{d_{PQ}} = \sqrt{(\mathbf{x}_2 - \mathbf{x}_1)^2 + (\mathbf{y}_2 - \mathbf{y}_1)^2}$$

2. The **mid-point between P and Q** is given by

   Midpoint of P & Q: $\mathbf{M(\overline{x}, \overline{y})}$

   $$\overline{\mathbf{x}} = \frac{\mathbf{x}_1 + \mathbf{x}_2}{2} \; ; \; \overline{\mathbf{y}} = \frac{\mathbf{y}_1 + \mathbf{y}_2}{2}$$

Example:

Given points: $\mathbf{P}(\mathbf{x}_1, \mathbf{y}_1) = \mathbf{P}(-6, -2)$; $\mathbf{Q}(\mathbf{x}_2, \mathbf{y}_2) = \mathbf{Q}(4, 8)$

Distance between P & Q: d_{PQ}

$$\begin{aligned}\mathbf{d}_{\mathbf{PQ}} &= \sqrt{(\mathbf{x}_2 - \mathbf{x}_1)^2 + (\mathbf{y}_2 - \mathbf{y}_1)^2} \\ &= \sqrt{([4] - [-6])^2 + ([8] - [-2])^2} \\ &= \sqrt{(10)^2 + (10)^2} \\ &= \sqrt{200} = 10\sqrt{2} \approx 14.142\end{aligned}$$

Midpoint of P & Q: $\mathbf{M}(\overline{\mathbf{x}}, \overline{\mathbf{y}})$

$$\overline{\mathbf{x}} = \frac{\mathbf{x}_1 + \mathbf{x}_2}{2} \; ; \; \overline{\mathbf{y}} = \frac{\mathbf{y}_1 + \mathbf{y}_2}{2}$$

$$\overline{\mathbf{x}} = \frac{[-6] + [4]}{2} = \frac{-2}{2} = -1$$

$$\overline{\mathbf{y}} = \frac{[-2] + [8]}{2} = \frac{6}{2} = 3$$

$\mathbf{M}(-1, 3)$