FUNctions – Introduction (Informal)

$$\left[\underset{\text{Allowable}}{\mathbf{x}} \rightarrow \overset{\mathbf{f}}{\rightarrow} \rightarrow \underset{\text{Unique}}{\mathbf{y}} = \mathbf{f}(\mathbf{x})\right]$$

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Basic FUNction Idea/Concept – For each “allowable input” there corresponds a “unique output”:

$$\textbf{Allowable Input} \overset{\textbf{implies}}{\Rightarrow} \textbf{Unique Output}$$

FUNction – Formula

$$\mathbf{y} = \underbrace{\overset{\textbf{Name}}{\overbrace{\mathbf{f}}}\,(\underset{\textbf{Input}}{\mathbf{x}})}_{\textbf{Output}} = \text{formula}$$

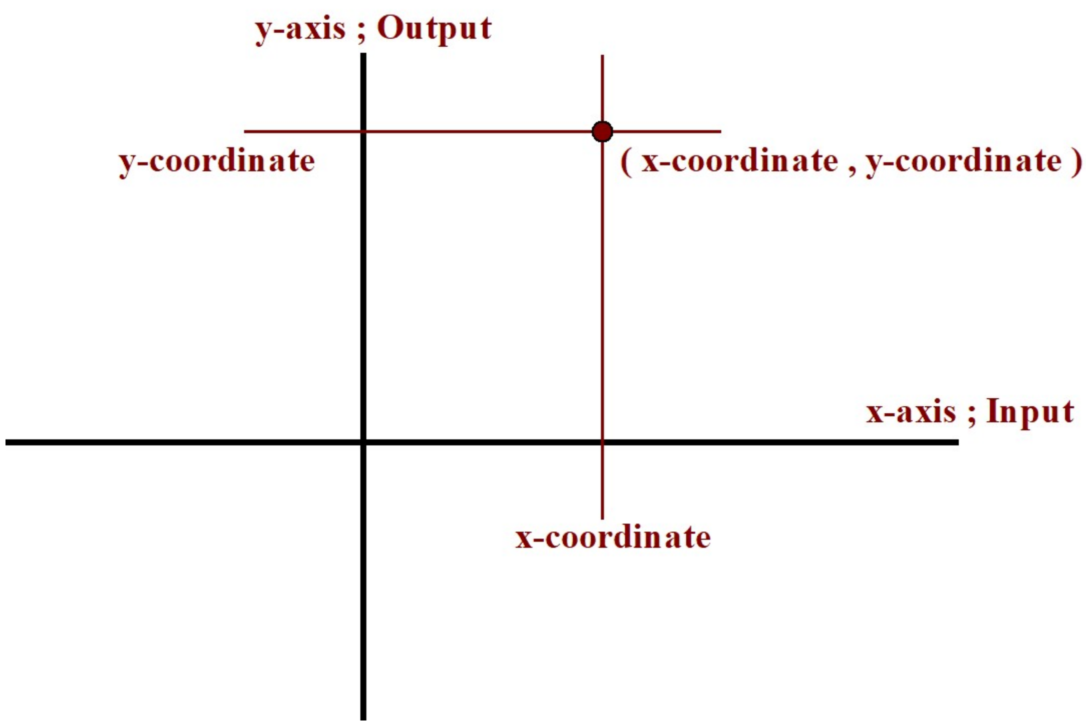
yields **ordered pairs:**

$$(\text{Input}\ ,\ \text{Output}) = (\mathbf{x} - \textbf{coordinate}, \mathbf{y} - \textbf{coordinate})$$
$$= (\mathbf{x}, \mathbf{y})$$

Note: A function can be defined many ways; for example, a “menu” is a function.

FUNction – Graph

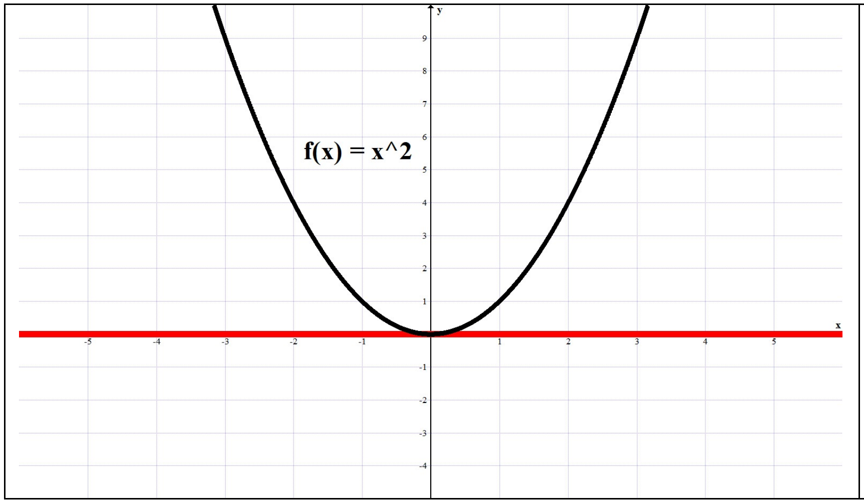
Graph of a FUNction is the collection of *all* these ordered pairs and is “plotted” in the number plane:

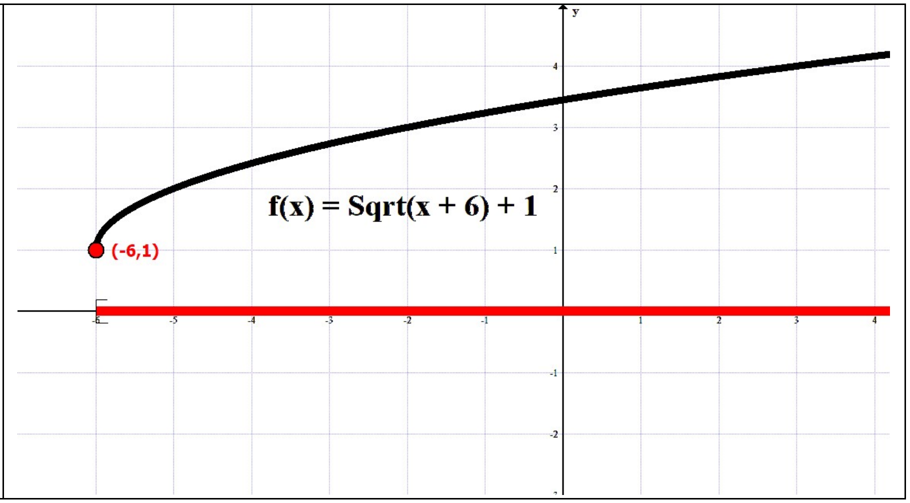


The functions we study will usually require us to “plot” an infinite number of points which is physically impossible. Therefore, we learn how to find the functions’ important properties which include important points, lines, and intervals, both x and y, that exhibit selected behavior. We will use some Basic and other functions, detailed later, to illustrate these properties. The *long term* goal is to learn how to find these properties analytically and use them, like puzzle pieces, to construct an accurate graph.

FUNction – Properties (Informal)

1. **Domain** (**Dom f**): Allowable inputs



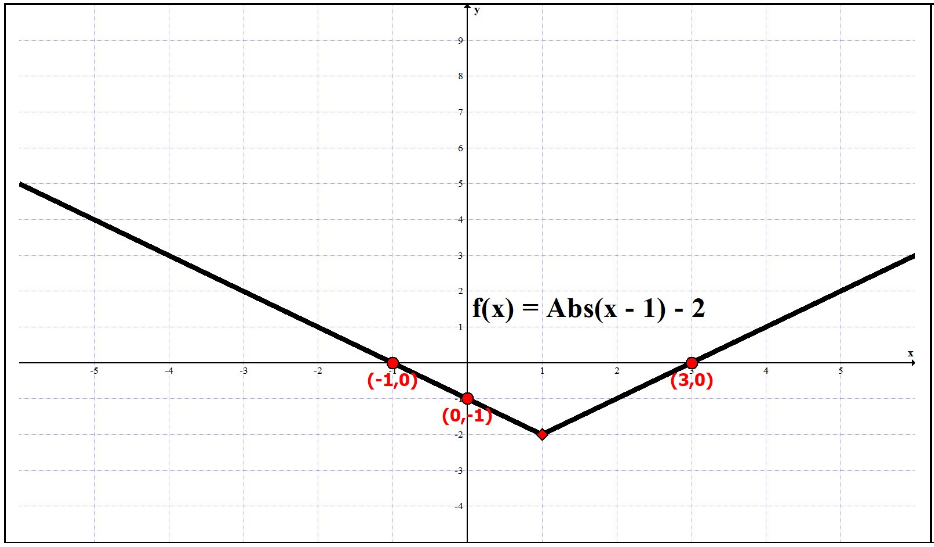


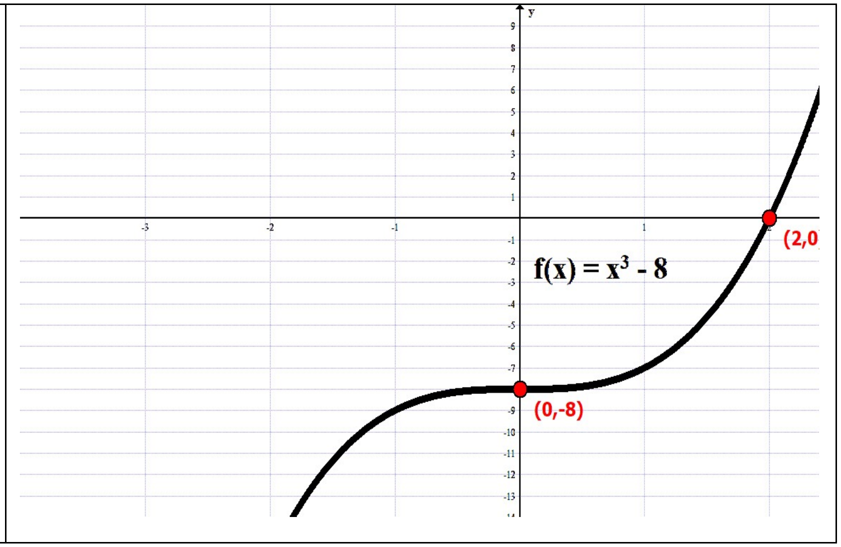
$\mathbf{Dom\,f} = \mathbb{R}_{\mathbf{x}} = (-\infty, +\infty)$

$\mathbf{Dom\,f} = [-6, +\infty)$

2. **Intercept POINTS:**

   a. y-intercept POINT: (0,f(0)) when 0 is the domain … Action Verb: **Evaluate**
   b. x-intercept POINT(s): (x,0) if f(x) = 0 … Action Verb: **Solve** y = f(x) for x





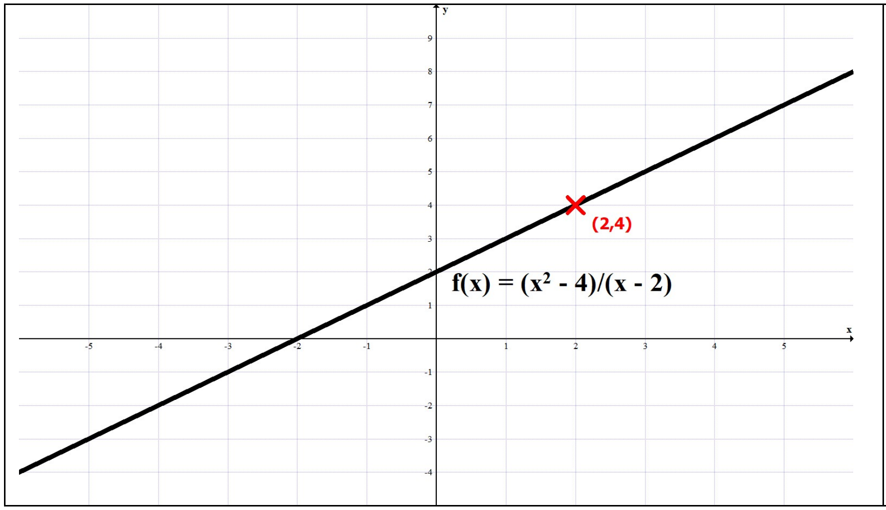
$\textbf{Intercept}_{\mathbf{y}} : (0,-1)$

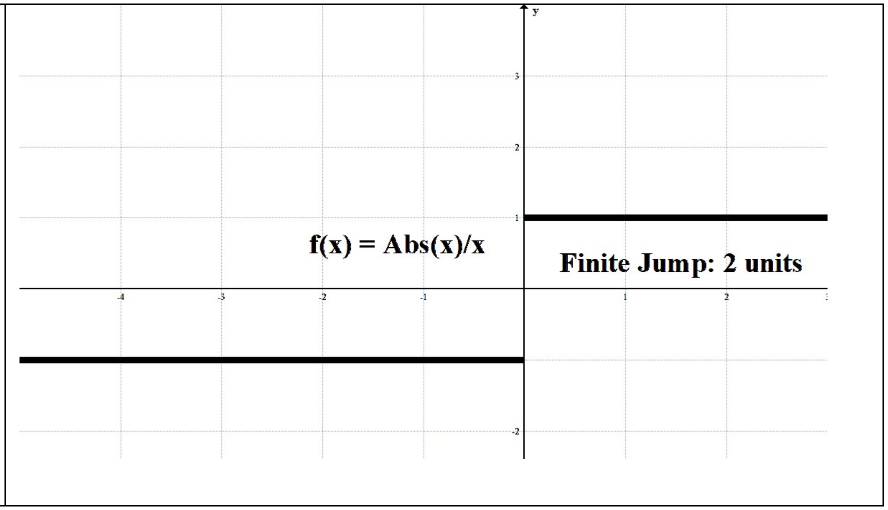
$\textbf{Intercepts}_{\mathbf{x}} : (-1,0);(3,0)$

$\textbf{Intercept}_{\mathbf{y}} : (0,-8)$

$\textbf{Intercept}_{\mathbf{x}} : (2,0)$

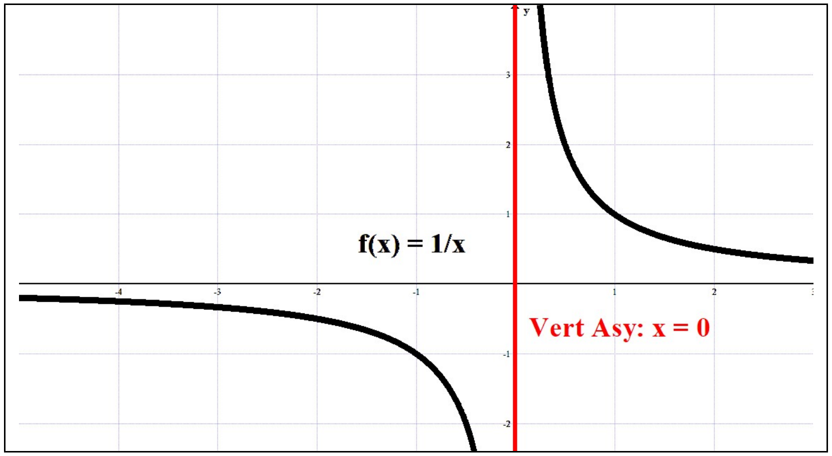
3. **Continuity** – NO breaks in graph
   Discontinuity – Breaks in graph
   a. **Hole**
   b. **Finite Jump** – stair step
   c. **Vertical Asymptote** – line: x = # where graph is getting “closer and closer” to





Hole at (2 , 4)

Finite Jump of 2 units at x = 0



The line x = 0 is a vertical asymptote

4. **Behavior at/toward infinity:**

   a. As the x-values get bigger and bigger in magnitude, what are the corresponding y-values doing?

      i. Choose x-values bigger and bigger $\left(\text{written } \mathbf{x} \to +\infty\right)$ & ask question: What are the corresponding y-values doing

      ii. Choose x-values smaller and smaller $\left(\text{written } \mathbf{x} \to -\infty \text{ or } -\infty \leftarrow \mathbf{x}\right)$ & ask question: What are the corresponding y-values doing

   b. y-values possibilities

      i. $\mathbf{y} \to \begin{cases} +\infty \text{ ; y-values go UP} \\ \text{or} \\ -\infty \text{ ; y-values go DOWN} \end{cases}$

      ii. $\mathbf{y} \to \#$; y-values get close to # ; line $\mathbf{y} = \#$ is called a horizontal asymptote

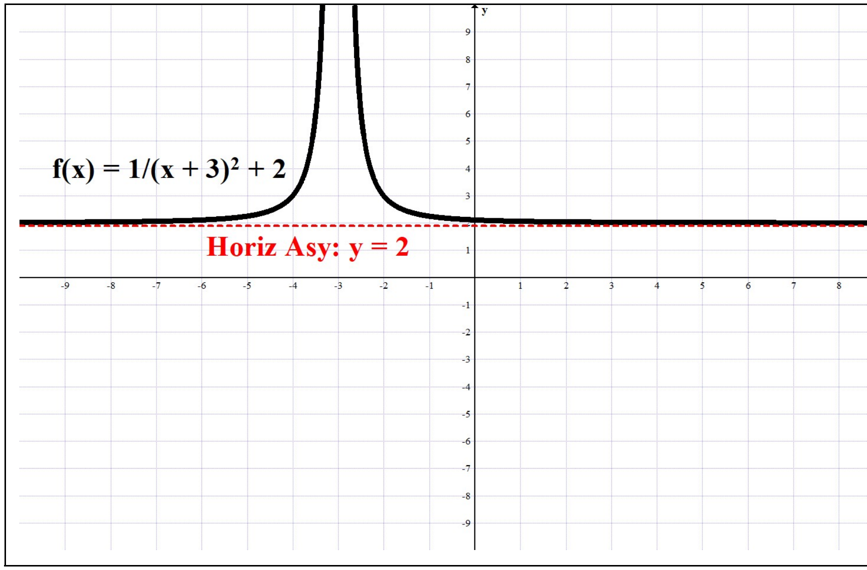
      iii. Something else happens

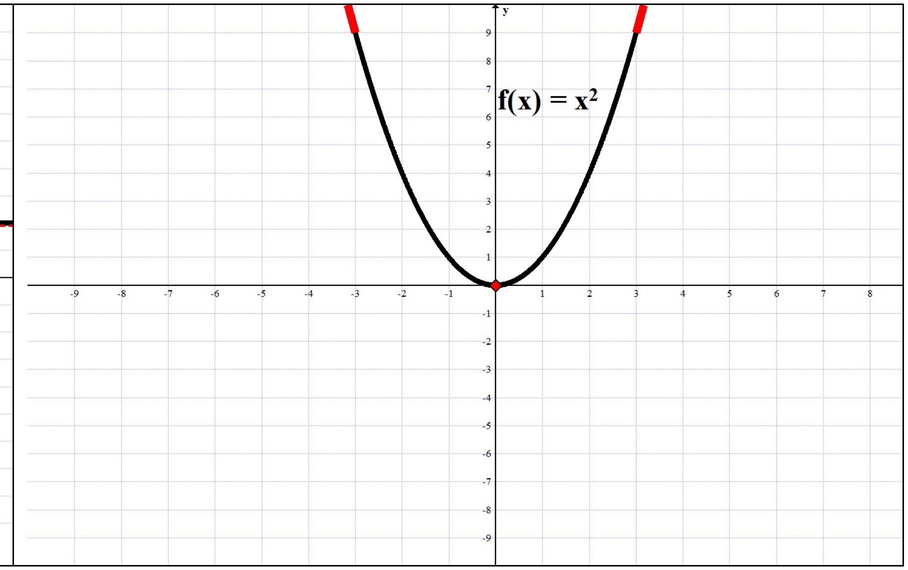
   c. Notations

      i. $\left.\begin{matrix} \mathbf{x} \to +\infty \\ \text{or} \\ \mathbf{x} \to -\infty \end{matrix}\right\} \Rightarrow \mathbf{y} \to \begin{cases} +\infty \text{ ; y-values go UP} \\ \text{or} \\ -\infty \text{ ; y-values go DOWN} \end{cases}$

   d.

      i. $\left.\begin{matrix} \mathbf{x} \to +\infty \\ \text{or} \\ \mathbf{x} \to -\infty \end{matrix}\right\} \Rightarrow \mathbf{y} \to \#$



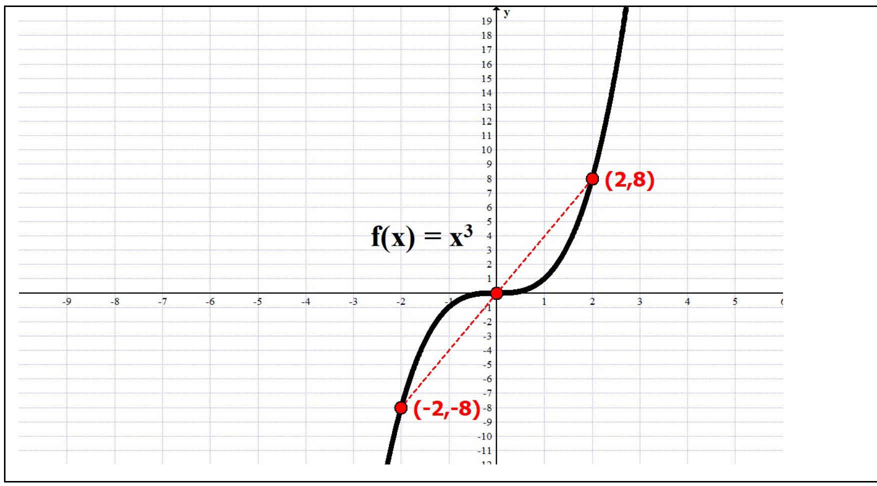


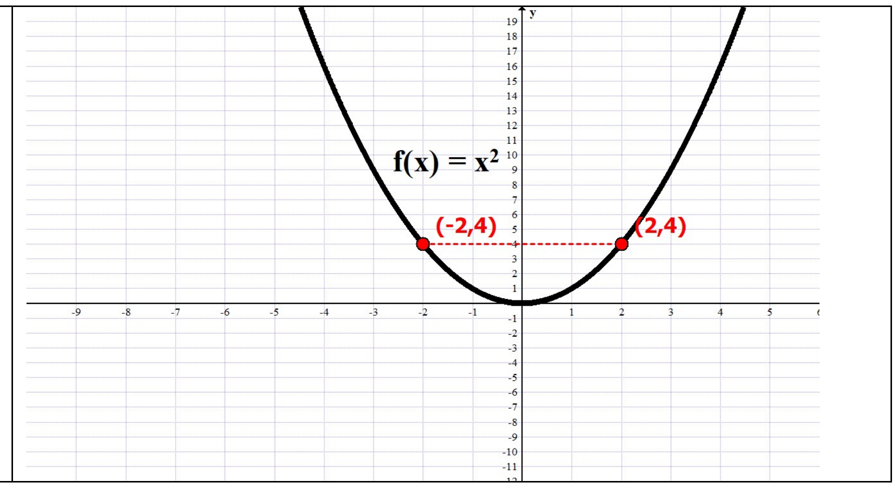
The line y = 2 is a horizontal asymptote

$|\mathbf{x}| \to \pm\infty \Rightarrow \mathbf{f}(\mathbf{x}) \to +\infty$

5. **Symmetry:**

   a. **Odd**: $\mathbf{f(-x)} = \overset{\text{Manipulate}}{\ldots} = -\mathbf{f(x)}$; graph is symmetric with respect to origin: (0,0)
   b. **Even**: $\mathbf{f(-x)} = \overset{\text{Manipulate}}{\ldots} = \mathbf{f(x)}$; graph is symmetric with respect to the y-axis: y = 0
   c. **Neither** even or odd



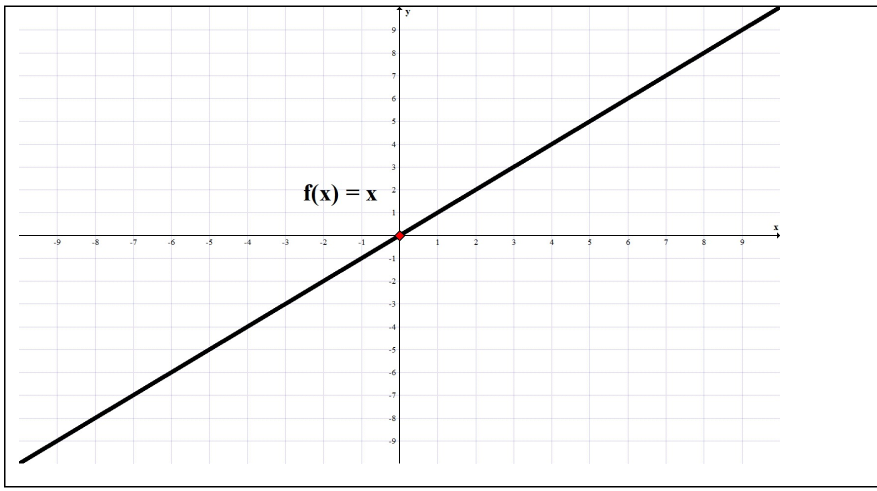


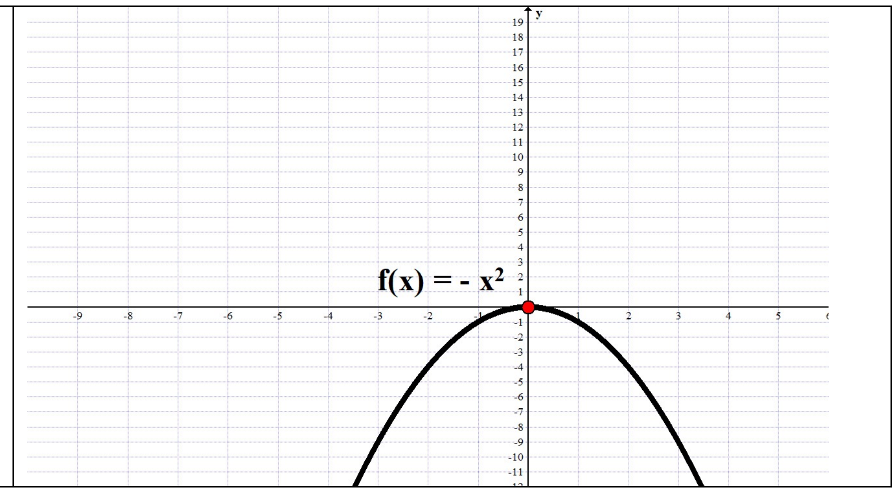
Graph symmetric wrt origin: (0 , 0)**: Odd**

Graph symmetric wrt y-axis: x = 0**: Even**

6. **Increasing/Decreasing:**
   a. **Increasing (Inc f):**
      x-values → Right ; y-values going UP:
      x-values → Right $\Rightarrow y = f(x) \nearrow$

   b. **Decreasing (Dec f):**
      x-values → Right ; y-values going DOWN:
      x-values → Right $\Rightarrow y = f(x) \searrow$



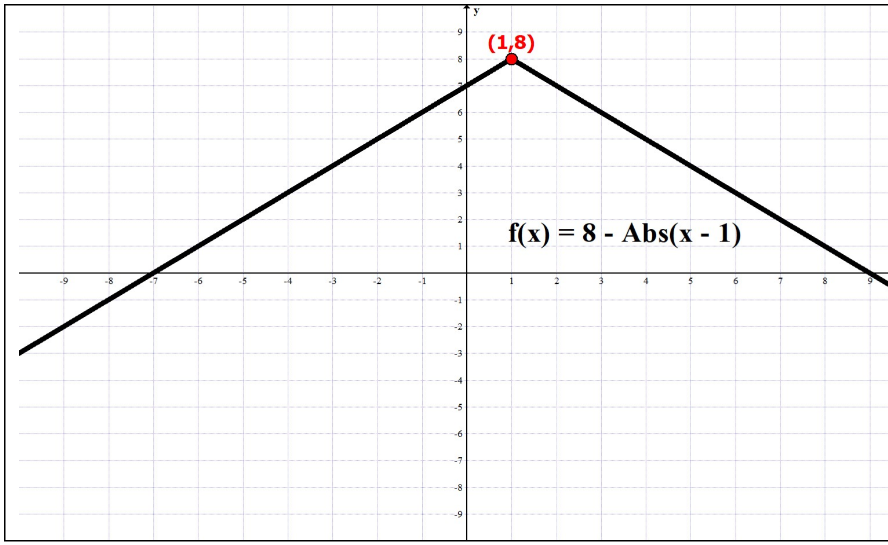


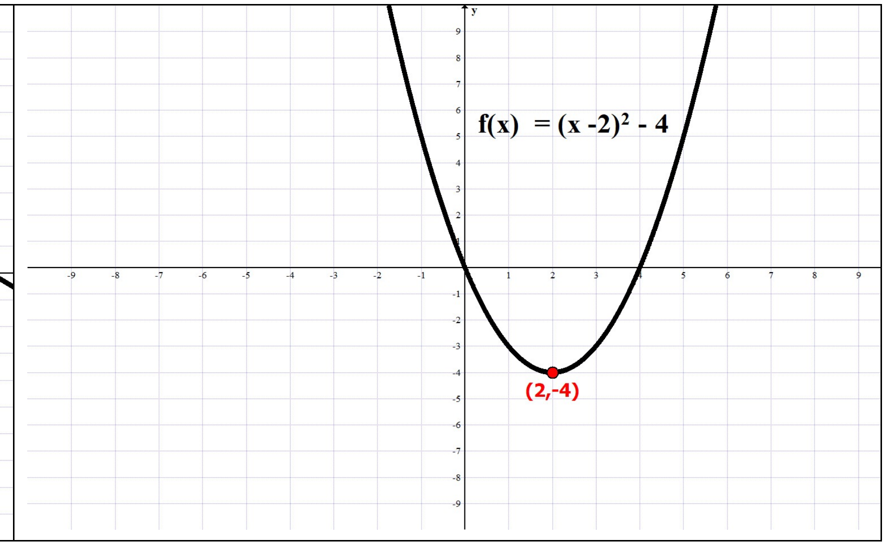
$$-\infty < \mathbf{x} < +\infty \Rightarrow \mathbf{f}(\mathbf{x}) \nearrow$$

$$-\infty < \mathbf{x} \le 0 \Rightarrow \mathbf{f}(\mathbf{x}) \nearrow$$
$$0 \le \mathbf{x} < +\infty \Rightarrow \mathbf{f}(\mathbf{x}) \searrow$$

7. **Relative Maximum/Minimum POINT(s):**

   a. **Relative Maximum POINT:** High point on the graph
   b. **Relative Minimum POINT:** Low point on the graph





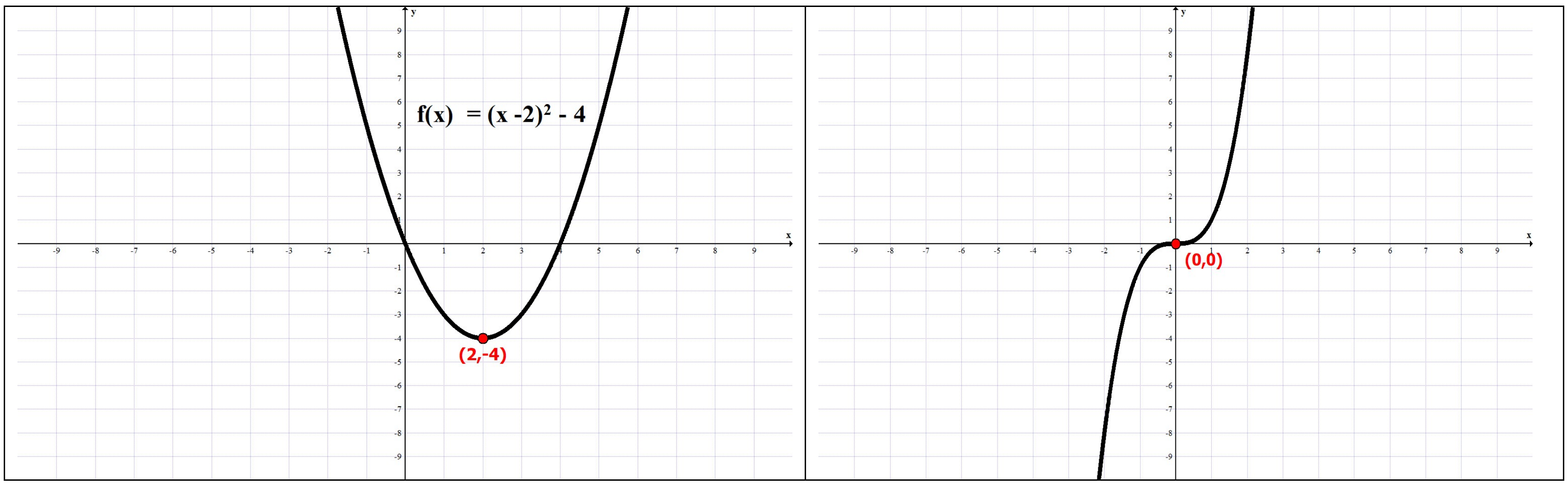
Relative Max Point: (1 , 8)

Relative Min Point: (2 , -4)

8. **Concavity:**

   a. **Concave UP (CU f):**
   x-values $\longrightarrow$ Right ; y-values are SMILING:
   x-values $\longrightarrow$ Right $\Rightarrow y = f(x)$ smiling

   b. **Concave DOWN (CD f):**
   x-values $\longrightarrow$ Right ; y-values are FROWNING:
   x-values $\longrightarrow$ Right $\Rightarrow y = f(x)$ frowning

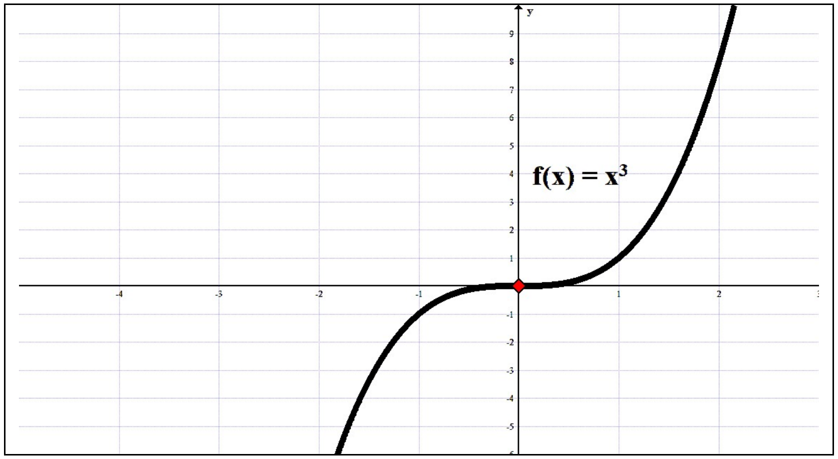


Concave Upward : $(-\infty,+\infty)_{\mathbf{x}}$

Concave Downward : $(-\infty,0]_{\mathbf{x}}$

Concave Upward : $[0,+\infty)_{\mathbf{x}}$

9. **Inflection POINT:** Point where graph changes from smiling to frowning or vice versa



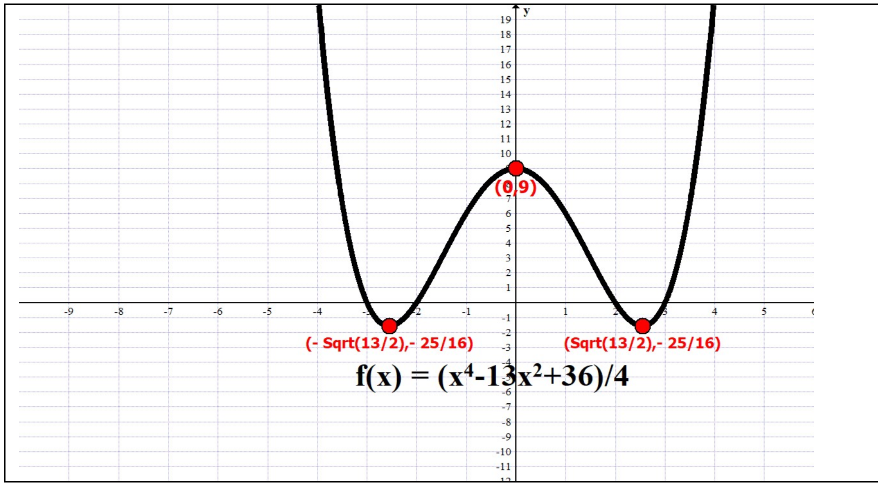
Inflection Point: $(0,0)_{\mathbf{x}}$

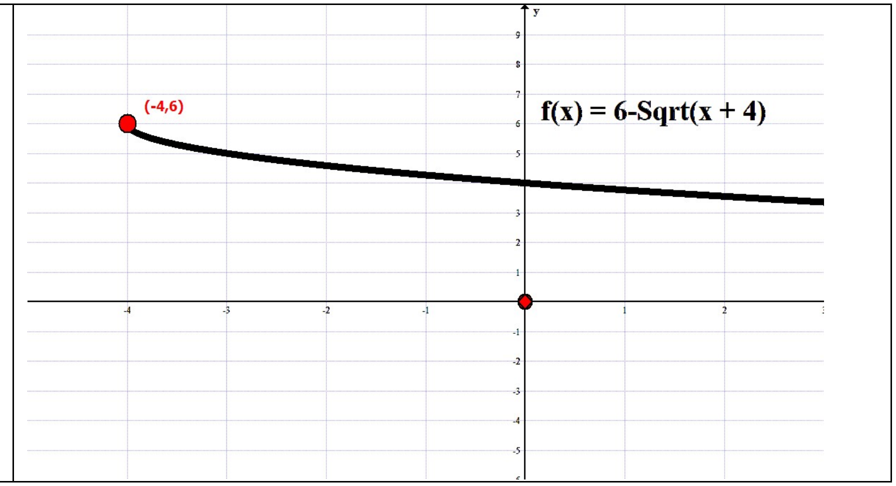
10. **Graph:** $\{(\mathbf{x},\mathbf{y}) \mid \mathbf{x} \in \mathbf{Dom\,f} \ \& \ \mathbf{y} = \mathbf{f}(\mathbf{x}) \in \mathbf{Rng\,f}\}$

    The *long term* goal is to find the first nine properties above, sketch an accurate graph, and then identify the final two (2) properties. Currently, we are just learning to identify the properties, called the **FUNction Summary Properties**.

11. **Absolute Maximum/Minimum POINT(s):**

a. **Absolute Maximum POINT:** Highest point on the graph
b. **Absolute Minimum POINT:** Lowest point on the graph





Relative Max Point: (0 , 9)

Relative Min Points: $\left(\pm\sqrt{\dfrac{13}{2}}, -\dfrac{25}{16}\right)$

Absolute Max Point: None

Absolute Min Points: $\left(\pm\sqrt{\dfrac{13}{2}}, -\dfrac{25}{16}\right)$

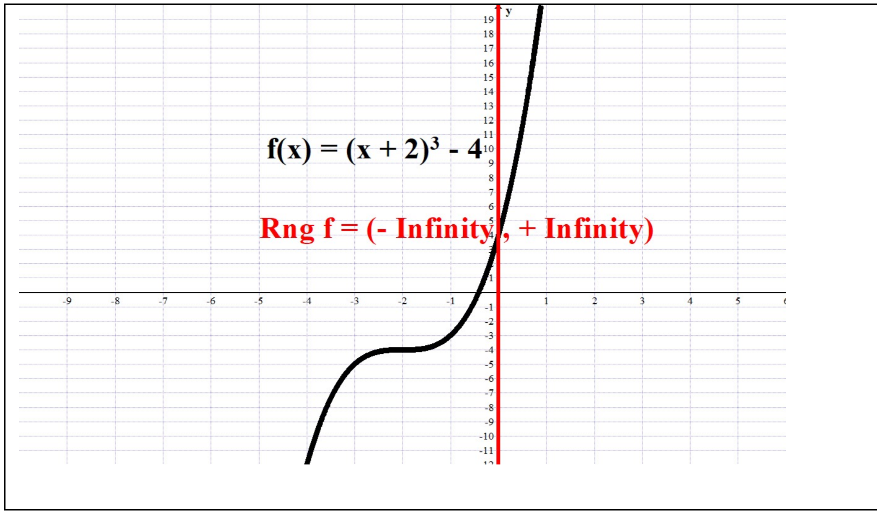
Relative Max Point: (-4 , 6)

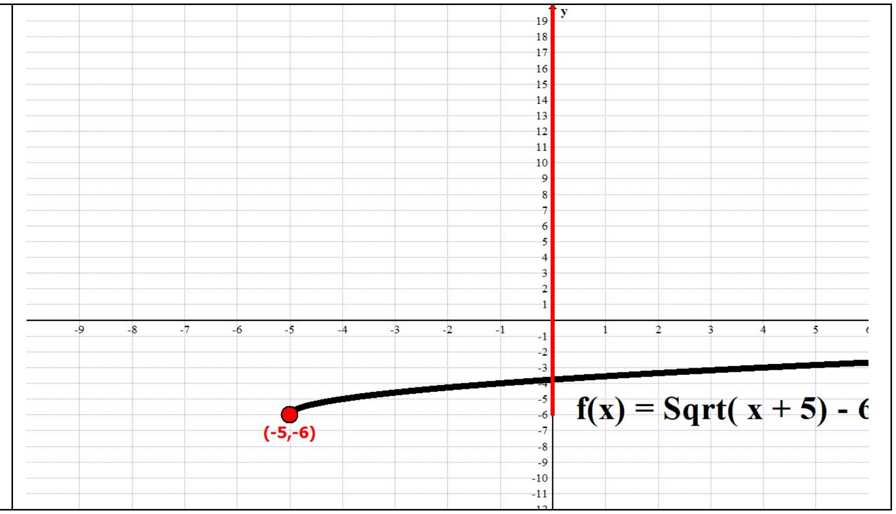
Relative Min Point: None

Absolute Max Point: $(-4, 6)$

Absolute Min Point: None

12.**Range (Rng f):** Unique outputs





Rng f = $(-\infty, +\infty)_y$

Rng f = $\left[-6, +\infty\right)_y$

13. **Additional Comments:** Anything else we observe!