FUNctions
Odd/Even

[Odd: f(-x) = - f(x): graph symmetric wrt (0,0)]

[Even: f(-x) = f(x): graph symmetric wrt x = 0]

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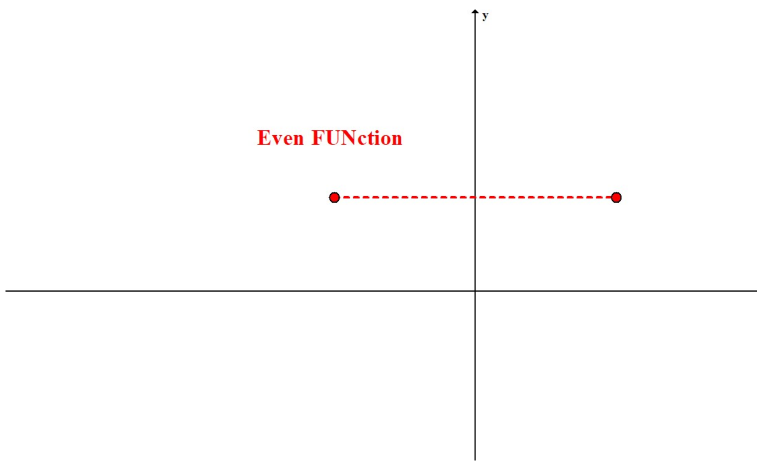
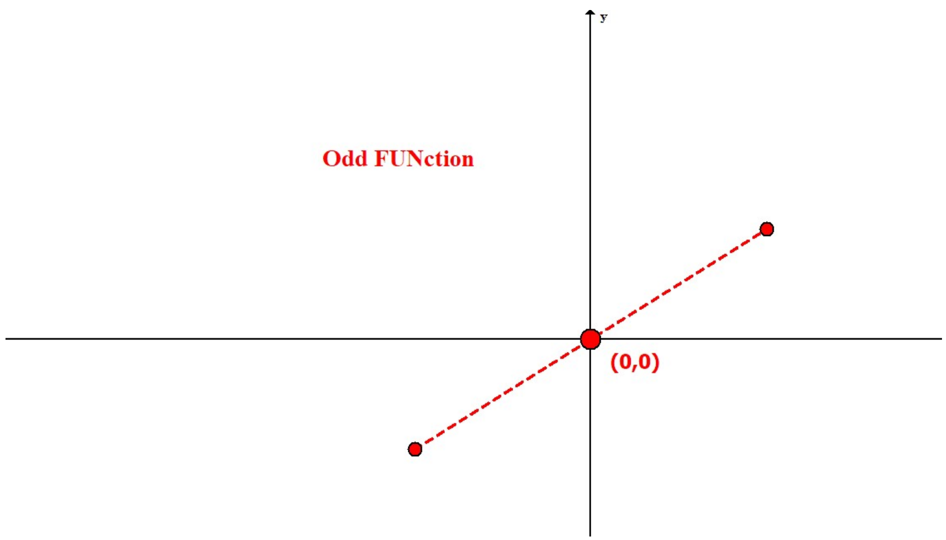
Definition: A function $\mathbf{f}$ is **even** if $\mathbf{f}(-\mathbf{x}) = \overset{\text{Manipulate}}{\ldots} = \mathbf{f}(\mathbf{x})$ for all $\mathbf{x} \in \mathbf{Dom\ f}$

Note: An *even* function will have a graph that is symmetric with respect to the y-axis.



Definition: A function $\mathbf{f}$ is **odd** if $\mathbf{f}(-\mathbf{x}) = \overset{\text{Manipulate}}{\ldots} = -\mathbf{f}(\mathbf{x})$ for all $\mathbf{x} \in \mathbf{Dom\ f}$

Note: An *odd* function will have a graph that is symmetric with respect to the Origin: $(0,0)$



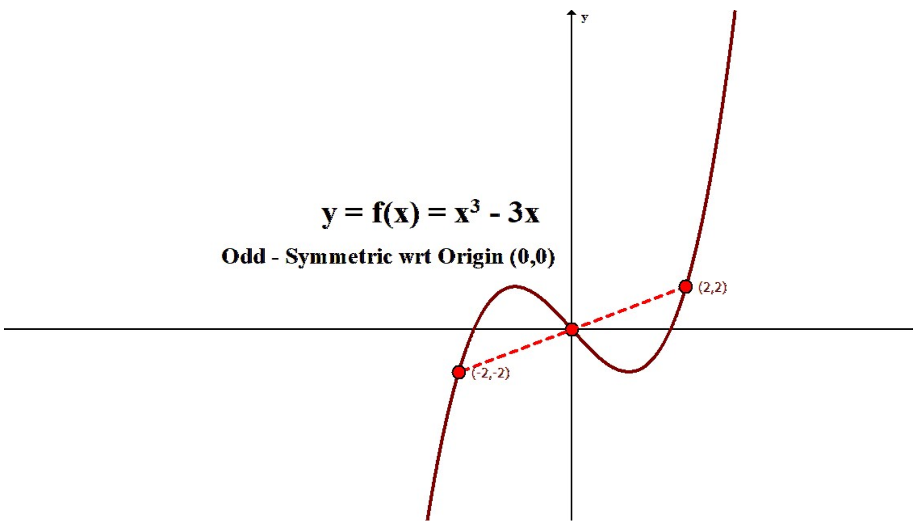
We will test the following functions to determine if they are odd, even or neither and then show their graphs to visually demonstrate the appropriate symmetry.

Example: Check the following functions to see if they are even, odd, or neither.

a. $\mathbf{f(x) = x^3 - 3x}$

We have $\mathbf{f(-x) = (-x)^3 - 3(-x) = -x^3 + 3x = -(x^3 - 3x) = -f(x)} \Rightarrow$ **ODD**

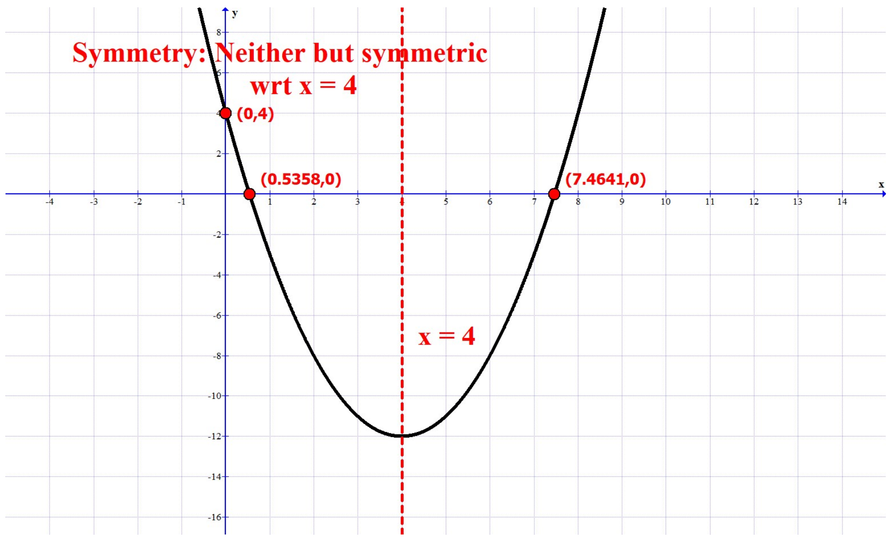
We know the graph is symmetric with respect to the Origin (0,0) as its graph below shows:



b. $\mathbf{f}(\mathbf{x}) = 4 - 8\mathbf{x} + \mathbf{x}^2$

We have $\mathbf{f}(-\mathbf{x}) = 4 - 8(-\mathbf{x}) + (-\mathbf{x})^2 = 4 + 8\mathbf{x} + \mathbf{x}^2 \neq \begin{cases} \mathbf{f}(\mathbf{x}) \\ -\mathbf{f}(\mathbf{x}) \end{cases} \Rightarrow$ **Neither**

We know the graph is NOT symmetric with respect to the Origin (0,0) or the y-axis (x = 0) as shown below:

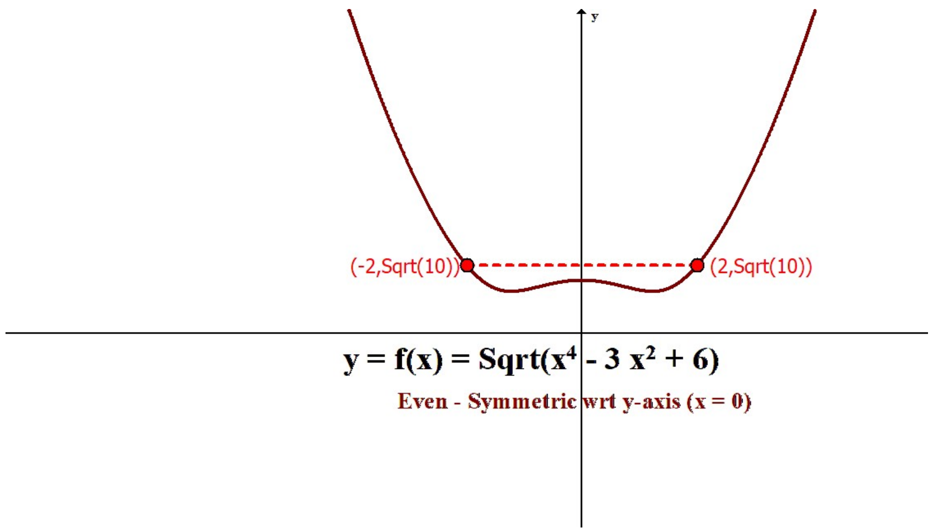


Note: This quadratic function is symmetric wrt the vertical line x = 4.

c. $\mathbf{f}(\mathbf{x}) = \sqrt{\mathbf{x}^4 - 3\mathbf{x}^2 + 6}$

We have $\mathbf{f}(-\mathbf{x}) = \sqrt{(-\mathbf{x})^4 - 3(-\mathbf{x})^2 + 6} = \sqrt{\mathbf{x}^4 - 3\mathbf{x}^2 + 6} = \mathbf{f}(\mathbf{x}) \Rightarrow$ **EVEN**

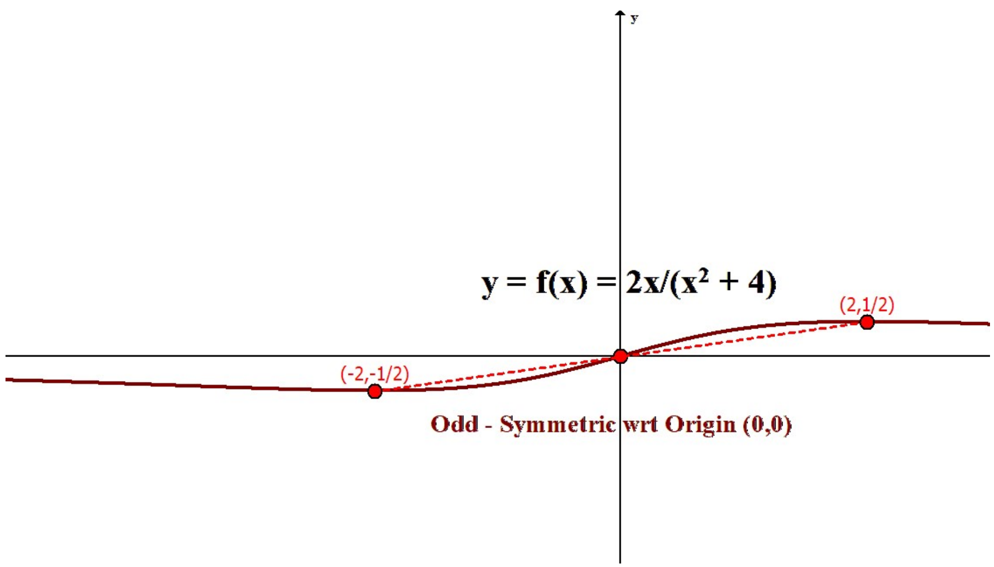
We know the graph is symmetric with respect to the y-axis (x = 0) as shown below:



d. $\mathbf{f}(\mathbf{x}) = \dfrac{2\mathbf{x}}{\mathbf{x}^2 + 4}$

We have $\mathbf{f}(-\mathbf{x}) = \dfrac{2(-\mathbf{x})}{(-\mathbf{x})^2 + 4} = \dfrac{-2\mathbf{x}}{\mathbf{x}^2 + 4} = -\dfrac{2\mathbf{x}}{\mathbf{x}^2 + 4} = -\mathbf{f}(\mathbf{x}) \Rightarrow \mathbf{ODD}$

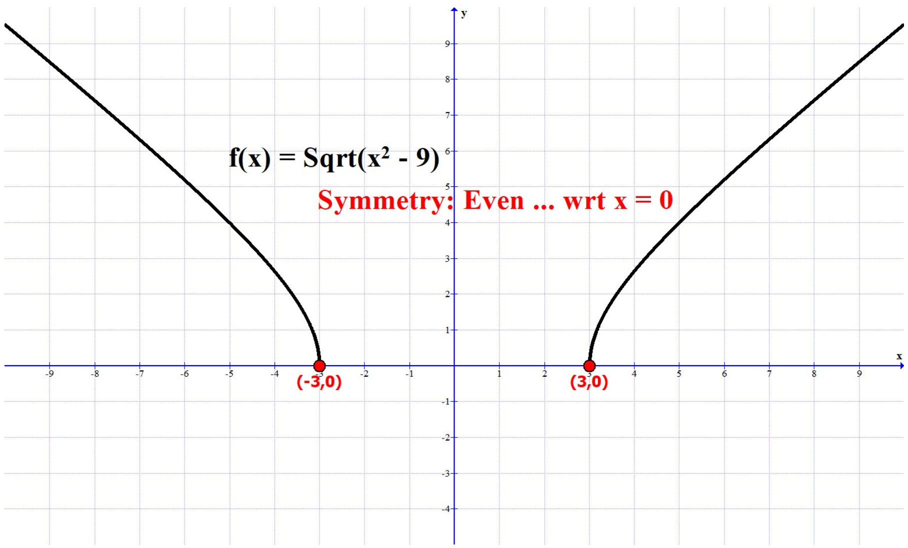
We know the graph is symmetric with respect to the Origin (0,0) as shown below:



e. $\mathbf{f}(\mathbf{x}) = \sqrt{\mathbf{x}^2 - 9}$

We have $\mathbf{f}(-\mathbf{x}) = \sqrt{(-\mathbf{x})^2 - 9} = \sqrt{\mathbf{x}^2 - 9} = \mathbf{f}(\mathbf{x}) \Rightarrow \mathbf{EVEN}$

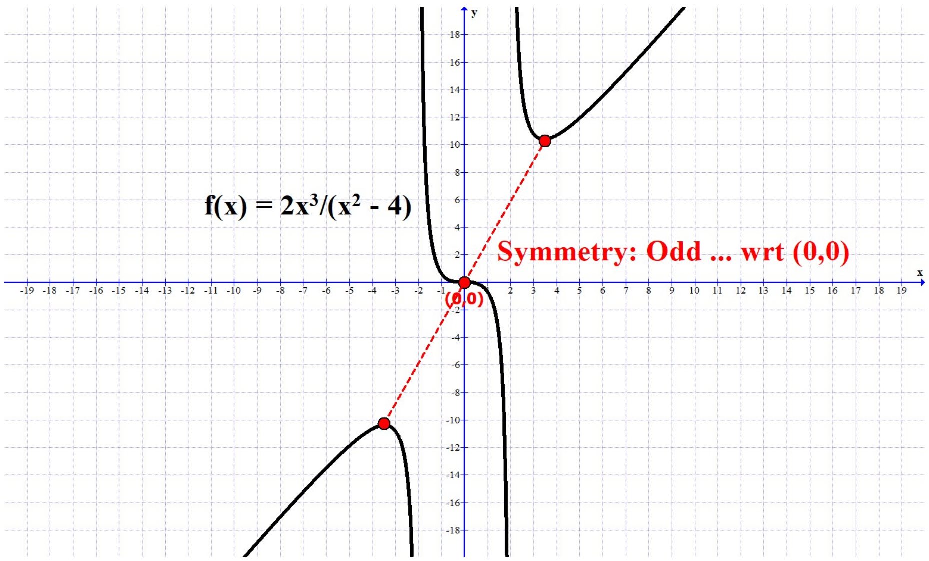
We know the graph is symmetric with respect to the y-axis as shown below:



f. $\mathbf{f}(\mathbf{x}) = \dfrac{2\mathbf{x}^3}{\mathbf{x}^2 - 4}$

We have $\mathbf{f}(-\mathbf{x}) = \dfrac{2(-\mathbf{x})^3}{(-\mathbf{x})^2 - 4} = -\dfrac{2\mathbf{x}^3}{\mathbf{x}^2 - 4} = -\mathbf{f}(\mathbf{x}) \Rightarrow \mathbf{ODD}$

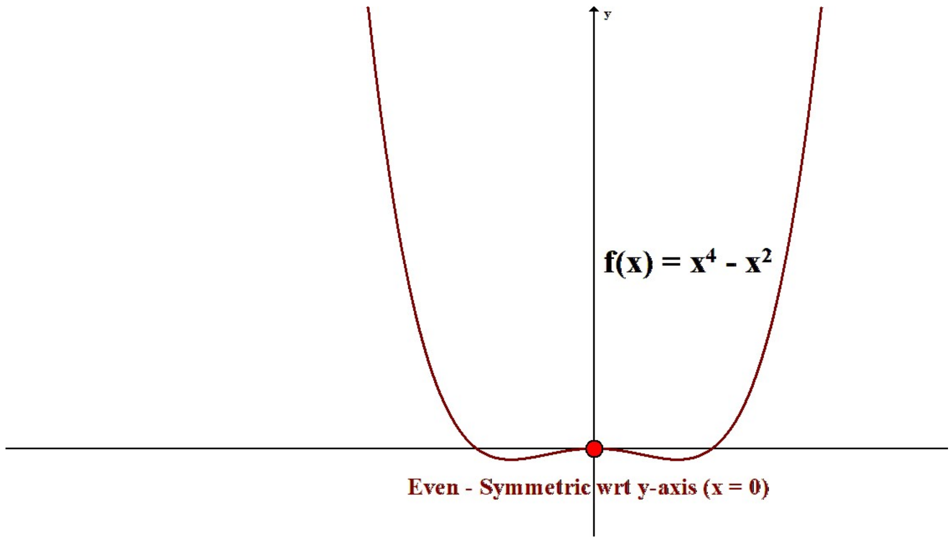
We know the graph is symmetric with respect to Origin (0,0) as shown below:



g. $\mathbf{f(x) = x^4 - 2x^2}$

We have $\mathbf{f(-x) = (-x)^4 - (-x)^2 = x^4 - x^2 = f(x) \Rightarrow EVEN}$

We know the graph is symmetric with respect to the y-axis as shown below:



h. $\mathbf{f}(\mathbf{x}) = 3 - 5\mathbf{x}^3 + 3\mathbf{x}^5$

We have $\mathbf{f}(-\mathbf{x}) = 3 - 5(-\mathbf{x})^3 + 3(-\mathbf{x})^5 = 3 + 5\mathbf{x}^3 - 3\mathbf{x}^5 \neq \begin{cases} \mathbf{f}(\mathbf{x}) \\ -\mathbf{f}(\mathbf{x}) \end{cases} \Rightarrow$ **Neither**

We know the graph is NOT even or odd as shown below: