Linear Functions - Equations of Lines

[y = f(x) = m x + b]

Slope (m) **y-intercept** (b)

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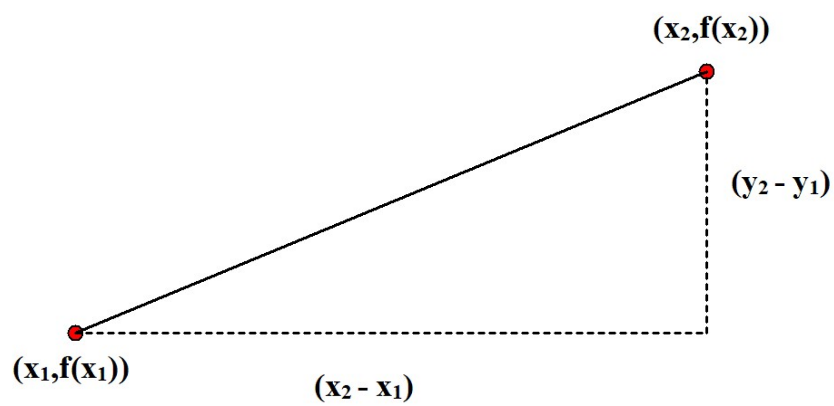
Linear Function Form (Slope & y-Intercept): $\mathbf{y} = \mathbf{f}(\mathbf{x}) = \mathbf{mx} + \mathbf{b}$

If $\mathbf{x} = 0 \Rightarrow \mathbf{y} = \mathbf{f}(0) = \mathbf{m} * 0 + \mathbf{b} = \mathbf{b} \Rightarrow (0, \mathbf{b})$ is the y-intercept point.

The letter **m** represents the "slope" of the "line" formed by $\mathbf{y} = \mathbf{f}(\mathbf{x}) = \mathbf{mx} + \mathbf{b}$. For if we pick two different points $(\mathbf{x}_1, \mathbf{y}_1)$ & $(\mathbf{x}_2, \mathbf{y}_2)$ on the graph of $\mathbf{f}$ and find $\frac{\text{Change in the y-values}}{\text{Change in the x-values}}$, we obtain

$$\begin{aligned} \frac{\text{Change in the y-values}}{\text{Change in the x-values}} &= \frac{\mathbf{f}(\mathbf{x}_2) - \mathbf{f}(\mathbf{x}_1)}{\mathbf{x}_2 - \mathbf{x}_1} \\ &= \frac{(\mathbf{mx}_2 + \mathbf{b}) - (\mathbf{mx}_1 + \mathbf{b})}{\mathbf{x}_2 - \mathbf{x}_1} \\ &= \frac{\mathbf{m}(\mathbf{x}_2 - \mathbf{x}_1)}{(\mathbf{x}_2 - \mathbf{x}_1)} \\ &= \mathbf{m} \end{aligned}$$

Slope



So, for any two points, the $\frac{\text{Change in the y-values}}{\text{Change in the x-values}}$ is ALWAYS the same constant **m**. Therefore, its graph is a straight line with slope **m**. If $\mathbf{m} = 0$, we obtain a horizontal line: $\mathbf{y} = \mathbf{b}$

Consider $\mathbf{y} = \mathbf{f}(\mathbf{x}) = \mathbf{mx} + \mathbf{b} = -\frac{3}{2}\mathbf{x} + 4$. We'll now find four (4) important properties that **f** possesses.

Properties:

1. Domain: $\textbf{Dom } \mathbf{f} = \mathbb{R}_{\mathbf{x}}$; there are no real numbers to reject.

2. Intercepts:

   a. $\mathbf{y}$: Set $\mathbf{x} = 0$

      $\mathbf{f}(0) = 4 \Rightarrow (0,4) = (0,\mathbf{b})$; y-intercept point

   b. $\mathbf{x}$: Set $\mathbf{y} = \mathbf{f}(\mathbf{x}) = 0$

      $0 \overset{\textbf{SET}}{=} \mathbf{y} = \mathbf{f}(\mathbf{x}) = -\frac{3}{2}\mathbf{x} + 4$

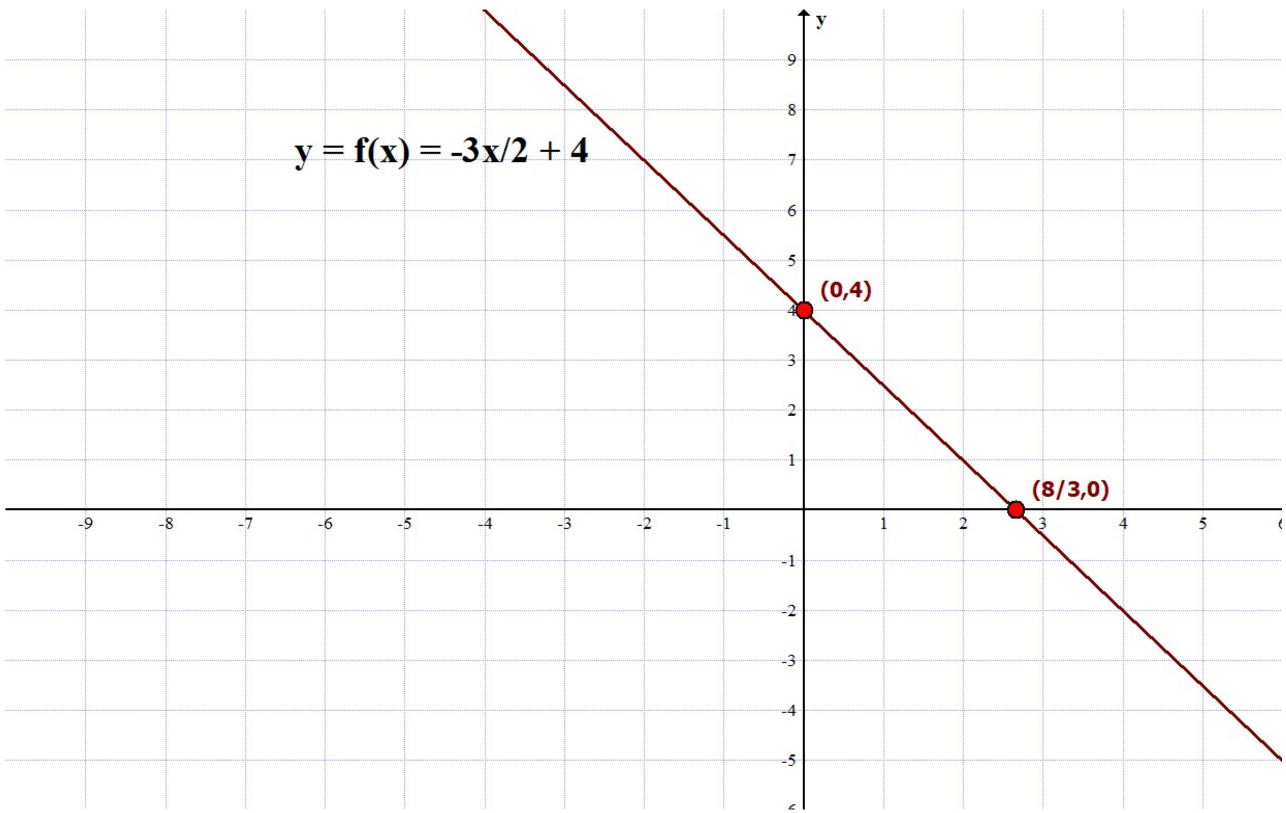
      $\frac{3}{2}\mathbf{x} = 4$

      $\mathbf{x} = \frac{8}{3} \Rightarrow \left(\frac{8}{3}, 0\right)$; **x**-intercept point

3. Slope: $\mathbf{m} = -\frac{3}{2}$

4. Range: $\textbf{Range } \mathbf{f} = \mathbb{R}_{\mathbf{y}}$, note the **range** is the projection of the graph onto the y-axis.

Drawing a straight line through the two (2) intercept points, we obtain the graph of **f**:



Note: The slope is *negative* so the line is slanted downward.

Although a straight line may be represented by $\mathbf{y = mx + b}$, there are several other ways to represent a line:

Given two (2) points $\mathbf{P(x_1, y_1)}$ and $\mathbf{Q(x_2, y_2)}$ (so that the slope is $\mathbf{m = \frac{y_2 - y_1}{x_2 - x_1}}$) **or** one point $\mathbf{P(x_1, y_1)}$ and a slope $\mathbf{m}$, there are four (4) forms the equation of a straight line using these data can take:

1. **Standard Form**: $\mathbf{Ax + By = C}$
   a. Two Points: $\mathbf{A = y_2 - y_1}$; $\mathbf{B = x_1 - x_2}$; $\mathbf{C = x_2 y_1 - x_1 y_2}$
   b. Point & Slope: $\mathbf{A = m}$; $\mathbf{B = -1}$; $\mathbf{C = mx_1 - y_1}$
2. **Slope & y-Intercept Form**: $\mathbf{y = mx + b}$; $\mathbf{m = \frac{y_2 - y_1}{x_2 - x_1}}$; $\mathbf{b = y_1 - \frac{y_2 - y_1}{x_2 - x_1} x_1}$ (Linear Function Form)
3. **Point & Slope Form**: $\mathbf{y - y_1 = m(x - x_1)}$; $\mathbf{m = \frac{y_2 - y_1}{x_2 - x_1}}$ if two points are given
4. **Two Point Form**: $\mathbf{y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)}$

Example 01: Consider the straight line $4\mathbf{x}+7\mathbf{y}=24$. Put this line in "linear function form" and find the properties of the function $\mathbf{f}$.

Solution:

We first solve $4\mathbf{x}+7\mathbf{y}=24$ for $\mathbf{y}$:

Step	Equation	Reason
0	$4\mathbf{x}+7\mathbf{y}=24$	$y=?$
1	$7\mathbf{y}=24-4\mathbf{x}$	
2	$\mathbf{y}=\frac{24-4\mathbf{x}}{7}=-\frac{4}{7}\mathbf{x}+\frac{24}{7}$	
3	$\mathbf{y}=\mathbf{f}(\mathbf{x})=-\frac{4}{7}\mathbf{x}+\frac{24}{7}$ $=\mathbf{mx}+\mathbf{b}$	

Properties:

1. Domain: $\mathbf{Dom\ f}=\mathbb{R}_{\mathbf{x}}$

2. Intercepts:

   a. $\mathbf{y}$: Set $\mathbf{x}=0$

   $\mathbf{f}(0)=\frac{24}{7}\Rightarrow\left(0,\frac{24}{7}\right)=(0,\mathbf{b})$; y-intercept point

   b. $\mathbf{x}$: Set $\mathbf{y}=\mathbf{f}(\mathbf{x})=0$

$$0 \overset{\text{SET}}{=} \mathbf{y} = \mathbf{f}(\mathbf{x}) = -\frac{4}{7}\mathbf{x} + \frac{24}{7}$$

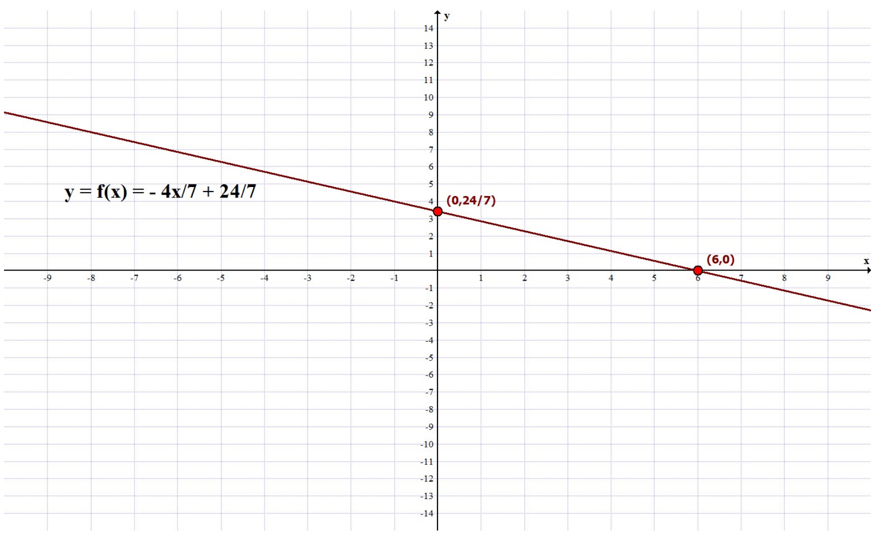
$$\frac{4}{7}\mathbf{x} = \frac{24}{7}$$

$$\mathbf{x} = \frac{24}{4} = 6 \Rightarrow (6,0) \text{ ; x-intercept point}$$

3. Slope: $\mathbf{m} = -\frac{4}{7}$

4. Range: $\textbf{Range f} = \mathbb{R}_{\mathbf{y}}$

Below, we have the graph of **f** :



Example 02: Consider the straight line defined by $\mathbf{m} = \frac{5}{3}$ and $\mathbf{P}(-4,7)$. Put this line in "linear function form" and find the properties of the function $\mathbf{f}$.

Solution:

Using the Point & Slope Form, we obtain:

Step	Calculation	Reason
0	$\mathbf{y}-\mathbf{y}_1=\mathbf{m}(\mathbf{x}-\mathbf{x}_1)$	Point-slope form
1	$\mathbf{y}-[7]=\left[\frac{5}{3}\right](\mathbf{x}-[-4])$	
2	$\mathbf{y}=\frac{5}{3}\mathbf{x}+\frac{20}{3}+7=\frac{5}{3}\mathbf{x}+\frac{41}{3}$	
3	$\mathbf{y}=\mathbf{f}(\mathbf{x})=\frac{5}{3}\mathbf{x}+\frac{41}{3}$ $=\mathbf{mx}+\mathbf{b}$	

Properties:

1. Domain: $\mathbf{Dom\ f}=\mathbb{R}_{\mathbf{x}}$

2. Intercepts:

    a. $\mathbf{y}$: Set $\mathbf{x}=0$

    $\mathbf{f}(0)=\frac{41}{3}\Rightarrow\left(0,\frac{41}{3}\right)=(0,\mathbf{b})$; y-intercept point

b. $\mathbf{x}$: Set $\mathbf{y}=\mathbf{f}(\mathbf{x})=0$

$$\mathbf{y}=\mathbf{f}(\mathbf{x})=\frac{5}{3}\mathbf{x}+\frac{41}{3}=0$$

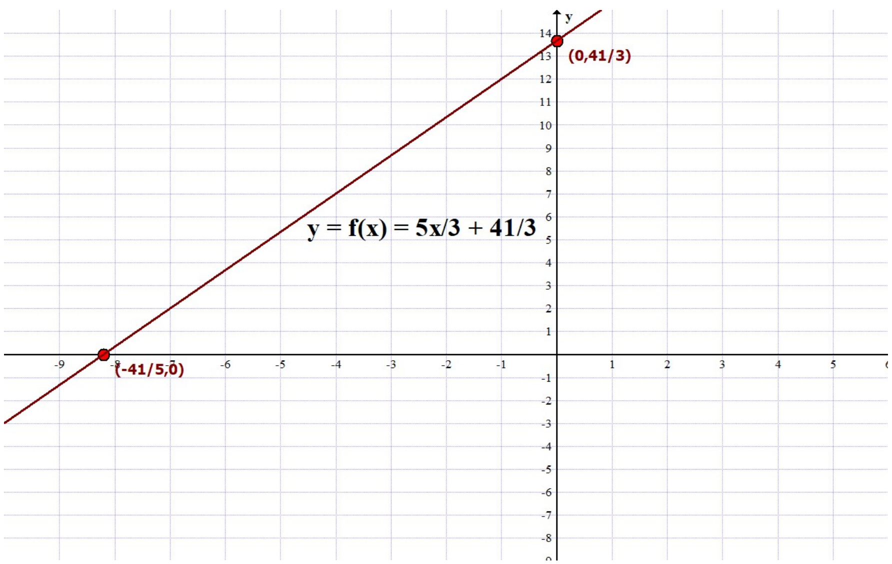
$$\frac{5}{3}\mathbf{x}=-\frac{41}{3}$$

$$\mathbf{x}=-\frac{41}{5}\Rightarrow\left(-\frac{41}{5},0\right)$$; x-intercept point

3. Slope: $\mathbf{m}=\frac{5}{3}$; given

4. Range: $\textbf{Range f}=\mathbb{R}_{\mathbf{y}}$

The graph is below:



Example 03: Consider the straight line defined by $\mathbf{P}(-3,2)$ and $\mathbf{Q}(5,9)$. Put this line in "linear function form" and find the properties of the function $\mathbf{f}$.

Solution:

The slope is $\mathbf{m}=\frac{\mathbf{y}_2-\mathbf{y}_1}{\mathbf{x}_2-\mathbf{x}_1}=\frac{[9]-[2]}{[5]-[-3]}=\frac{7}{8}$. Now, using the Point & Slope Form, we have:

Step	Calculation	Reason
0	$\mathbf{y}-\mathbf{y}_1=\mathbf{m}(\mathbf{x}-\mathbf{x}_1)$	Point-slope form
1	$\mathbf{y}-[2]=\left[\frac{7}{8}\right](\mathbf{x}-[-3])$	
2	$\mathbf{y}=\frac{7}{8}\mathbf{x}+\frac{21}{8}+2=\frac{7}{8}\mathbf{x}+\frac{37}{8}$	
3	$\mathbf{y}=\mathbf{f}(\mathbf{x})=\frac{7}{8}\mathbf{x}+\frac{37}{8}$	

Properties:

1. Domain: $\mathbf{Dom\ f}=\mathbb{R}_{\mathbf{x}}$

2. Intercepts:

   a. $\mathbf{y}$: Set $\mathbf{x}=0$

   $\mathbf{f}(0)=\frac{37}{8}\Rightarrow\left(0,\frac{37}{8}\right)=(0,\mathbf{b})$

b. $\mathbf{x}$: Set $\mathbf{y}=\mathbf{f}(\mathbf{x})=0$

$$\mathbf{y}=\mathbf{f}(\mathbf{x})=\frac{7}{8}\mathbf{x}+\frac{37}{8}\overset{\mathbf{SET}}{=}0$$

$$\frac{7}{8}\mathbf{x}=-\frac{37}{8}$$

$$\mathbf{x}=-\frac{37}{7}\Rightarrow\left(-\frac{37}{7},0\right)$$

3. Slope: $\mathbf{m}=\frac{7}{8}$

4. Range: $\mathbf{Range\ f}=\mathbb{R}_{\mathbf{y}}$

The graph is below: