Linear Functions - More Topics

[Lines: Vertical, Horizontal, parallel, perpendicular]

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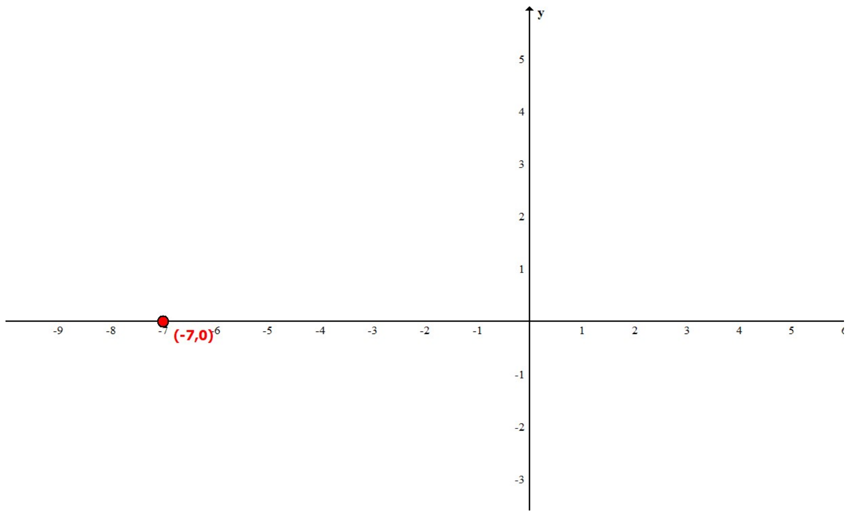
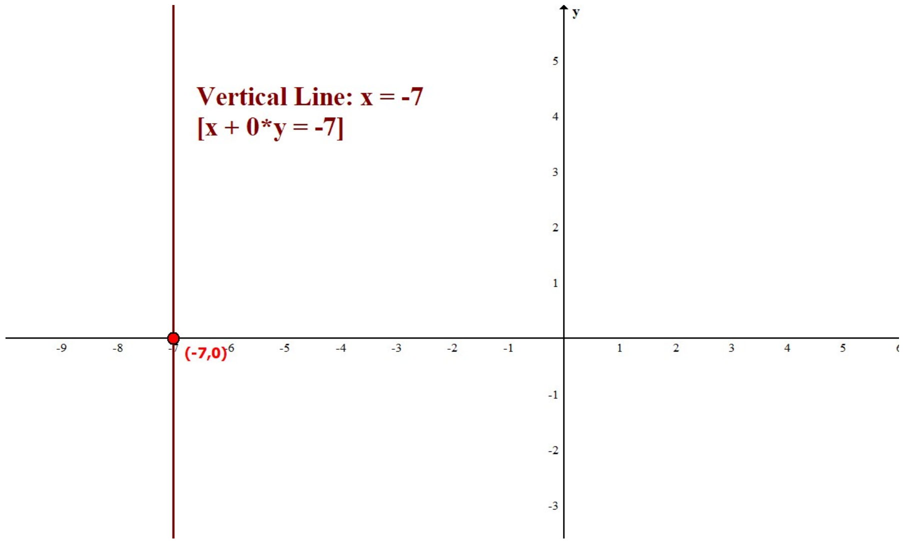
Vertical Lines: $x =$ Number

Example 01: Draw the graph of $x = -7$.

Solution: If we are in one dimension, $x = -7$ is just a point on the Horizontal Number Line:



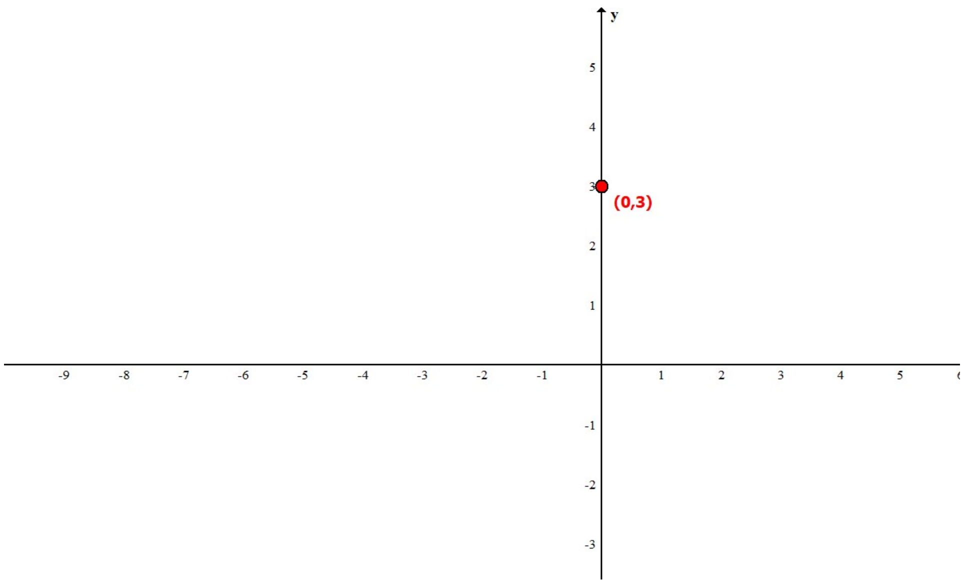
If we are in two dimensions, $x = -7$ can be written as $x + 0 * y = -7$ so that points on its graph are $(-7, y)$ where y can be *any* number on the Vertical Number Line.



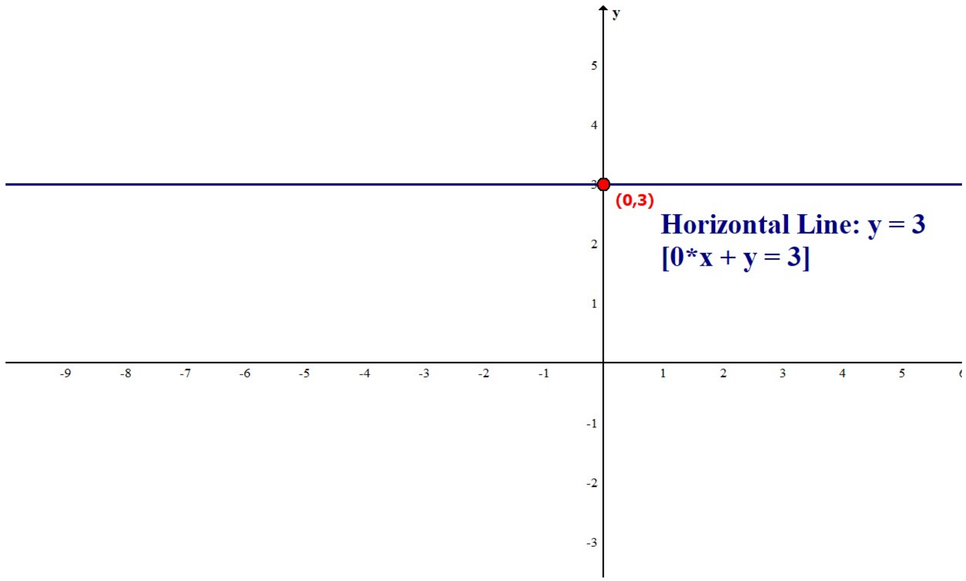
Horizontal Lines: $y =$ Number

Example 02: Draw the graph of $y = 3$.

Solution: If we are in one dimension, $y = 3$ is just a point on the Vertical Number Line:



If we are in two dimensions, $y = 3$ can be written as $0 * x + y = 3$ so that points on its graph are $(x, 3)$ where x can be *any* number on the Horizontal Number Line.



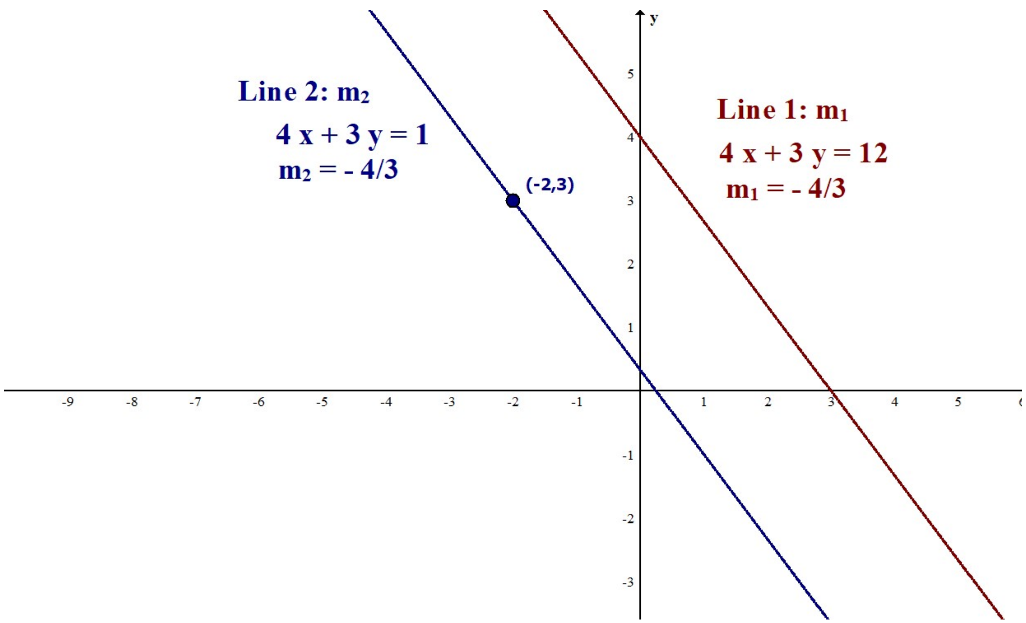
Parallel Lines: $l_1 // l_2 \Rightarrow m_1 = m_2$ obviously

Example 03: Given $\ell_1 : 4x + 3y = 12$, find the equation of the line ℓ_2 parallel to ℓ_1 that passes through the point $P(x_2, y_2) = P(-2, 3)$.

Solution: Solving $\ell_1 : 4x + 3y = 12$ for y yields $y = -\frac{4x}{3} + 4$ so that $m_2 = m_1 = -\frac{4}{3}$. Using the slope-intercept form $y = mx + b$, we obtain

$$y_2 = m_2 \ x_2 + b$$
$$b = y_2 - m_2 x_2$$
$$= 3 - \left(-\frac{4}{3}(-2) \right)$$
$$= \frac{1}{3}$$

Thus $\ell_2 : y = -\frac{4x}{3} + \frac{1}{3}$. The graphs are shown below:



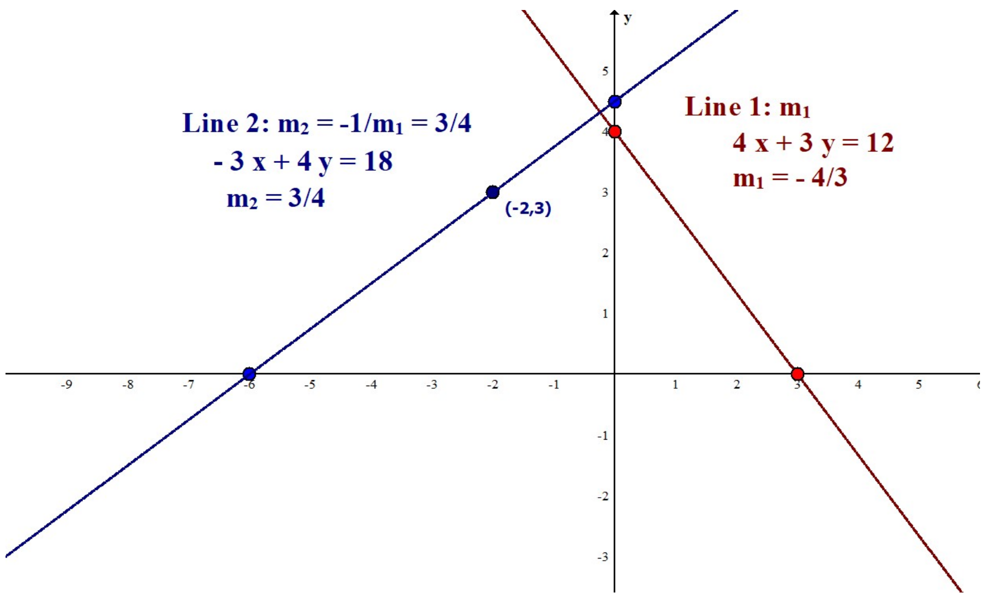
Perpendicular Lines: $l_1 \perp l_2 \Rightarrow m_1 * m_2 = -1$; this is NOT obvious but true: $m_2 = \frac{-1}{m_1}$ $(m_1 \neq 0)$

Example 04: Given $\ell_1 : 4x + 3y = 12$, find the equation of the line ℓ_2 perpendicular to ℓ_1 that passes through the point $P(x_2, y_2) = P(-2,3)$.

Solution: Solving $\ell_1 : 4x + 3y = 12$ for y yields $y = -\frac{4x}{3} + 4$ so that $m_2 = -\frac{1}{m_1} = \frac{3}{4}$. Using the slope-intercept form $y = mx + b$, we obtain

$$\begin{aligned} y_2 &= m_2\ x_2 + b \\ b &= y_2 - m_2 x_2 \\ &= 3 - \left(\frac{3}{4}(-2)\right) \\ &= 9/2 \end{aligned}$$

Thus $\ell_2 : y = \frac{3x}{4} + \frac{9}{2}$. The graphs are shown below:


Line 2: $m_2 = -1/m_1 = 3/4$
$-3x + 4y = 18$
$m_2 = 3/4$
(-2,3)
Line 1: m_1
$4x + 3y = 12$
$m_1 = -4/3$
y

Average Rate of Change: With respect to an Interval $[a,b]$

1. **Linear Functions – Straight Lines** $(y = f(x) = (\text{Slope}) * x + (\text{y-intercept}))$

$$\frac{\text{Change in y values}}{\text{Change in x values}} = \frac{\Delta y}{\Delta x} = \frac{f(b) - f(a)}{b - a} = \text{Slope}$$

Example 05: Find the average rate of change for $y = f(x) = 3x + 4$ on $[2,5]$ & $[-1,3]$

Solution:

$[2,5]$**:** We have

$$\frac{f(b) - f(a)}{b - a} = \frac{f(5) - f(2)}{5 - 2} = \frac{(15 + 4) - (6 + 4)}{3}$$
$$= \frac{9}{3} = 3$$

$[-1,3]$**:** We have

$$\frac{f(b) - f(a)}{b - a} = \frac{f(3) - f(-1)}{3 - (-1)} = \frac{(9 + 4) - (-3 + 4)}{4}$$
$$= \frac{12}{4} = 3$$

Note: We have already noted that the slope of a straight line is independent of which points are selected.

2. Other Functions – NOT Straight Lines $(y = f(x) = \text{Formula})$

$$\frac{\text{Change in y values}}{\text{Change in x values}} = \frac{\Delta y}{\Delta x} = \frac{f(b) - f(a)}{b - a}$$

Example 06: Find the average rate of change for $y = f(x) = x^2 - 3x + 4$ on $[1,3]$ & $[-2,5]$.
Solution:

$[1,3]$**:** We have

$$\frac{f(b) - f(a)}{b - a} = \frac{f(3) - f(1)}{3 - 1} = \frac{(9 - 9 + 4) - (1 - 3 + 4)}{2}$$
$$= \frac{4}{2} = 2$$

$[-2,5]$**:** We have

$$\frac{f(b) - f(a)}{b - a} = \frac{f(5) - f(-2)}{5 - (-2)} = \frac{(25 - 15 + 4) - (4 + 6 + 4)}{7}$$
$$= \frac{0}{2} = 0$$

Note: For non-linear functions, the average rate of change may be dependent upon which two (2) points are selected.

