

FUNctions – Quadratic

$$y = f(x) = ax^2 + bx + c ; a \neq 0$$

Five Point Method

$$\left[\begin{array}{c} \text{MATH by Wilson} \\ \text{Your Personal Mathematics Trainer} \\ \text{MathByWilson.com} \end{array} \right]$$

With a linear function, we can obtain its graph by drawing a straight line through any two (2) points on the line. For a quadratic function, we can usually find five (5) special points and then draw the graph. The outline below shows how to find these points:

Given

$$y = f(x) = ax^2 + bx + c ; a \neq 0$$

$$a = \underline{\hspace{2cm}}$$

$$b = \underline{\hspace{2cm}}$$

$$c = \underline{\hspace{2cm}}$$

find the following points and use them to draw the graph:

1. x-intercept points (usually 2 points)
2. y-intercept point
3. symmetry point
4. vertex

a. **Domain:** Allowable x-values

$$\text{Domain} = \mathbb{R}_x = (-\infty, +\infty)_x$$

b. **POINTS #1 & #2:** Max of two x-intercept points

$$\text{Set } D = b^2 - 4ac$$

- (1) $D < 0$: NO x-intercept point
- (2) $D = 0$: ONE x-intercept point
- (3) $D > 0$: TWO x-intercept points

x-intercepts POINTS(s): $\left(\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, 0 \right)$

c. **POINT #3:**

y-intercept POINT: $(0, c)$

d. **POINT #4:**

Symmetry POINT: $\left(-\frac{b}{a}, c \right)$; also called “cheap point”

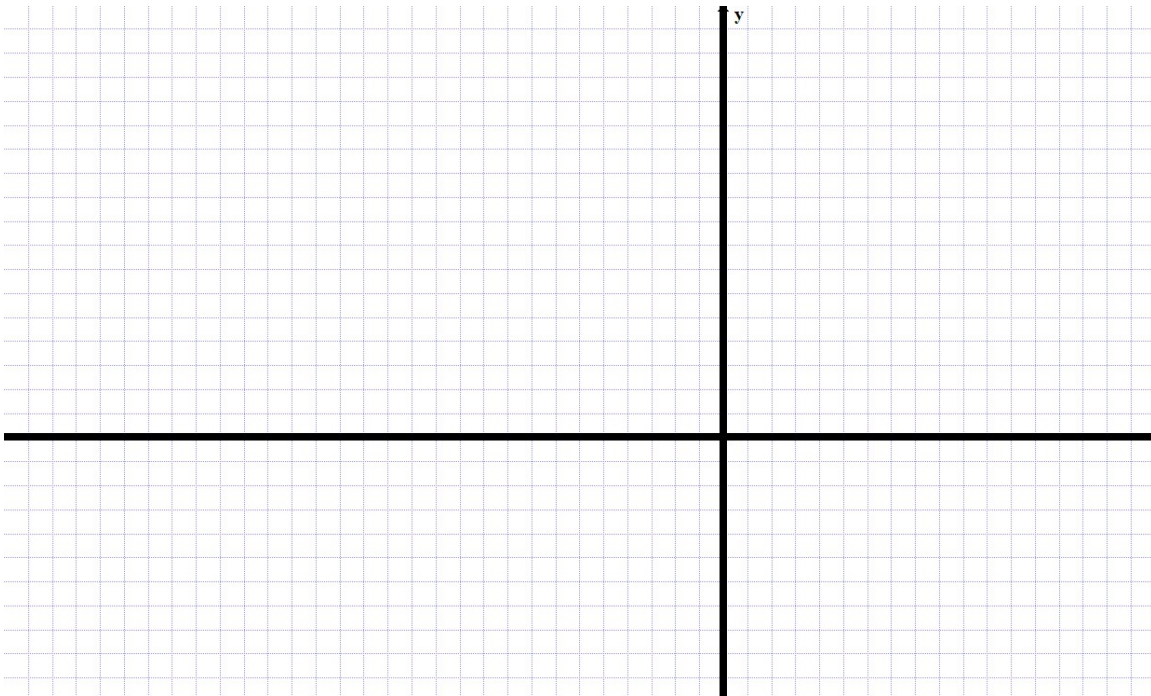
e. **POINT #5:**

Vertex POINT: $\left(-\frac{b}{2a}, \frac{4ac - b^2}{4a} \right)$

Absolute Maximum point if $a < 0$

Absolute Minimum point if $a > 0$

f. **Graph:** PLOT the five (5) points (Max) and sketch the graph



g. **Positive:** The x-values where $y > 0$

Negative: The x-values where $y < 0$

Assuming two (2) x-intercept points, we have

(1) If $a > 0$

$$\text{Positive} = \left(-\infty, \frac{-b - \sqrt{b^2 - 4ac}}{2a} \right] \cup \left[\frac{-b + \sqrt{b^2 - 4ac}}{2a}, +\infty \right)$$

$$\text{Negative} = \left[\frac{-b - \sqrt{b^2 - 4ac}}{2a}, \frac{-b + \sqrt{b^2 - 4ac}}{2a} \right]$$

(2) If $a < 0$

$$\text{Positive} = \left[\frac{-b - \sqrt{b^2 - 4ac}}{2a}, \frac{-b + \sqrt{b^2 - 4ac}}{2a} \right]$$

$$\text{Negative} = \left(-\infty, \frac{-b - \sqrt{b^2 - 4ac}}{2a} \right] \cup \left[\frac{-b + \sqrt{b^2 - 4ac}}{2a}, +\infty \right)$$

h. Range: Allowable y-values

$$\text{Range} = \left(-\infty, \frac{4ac - b^2}{4a} \right]_y \text{ if } a < 0$$

$$\text{Range} = \left[\frac{4ac - b^2}{4a}, +\infty \right)_y \text{ if } a > 0$$