FUNctions – Quadratic

$$\mathbf{y} = \mathbf{f}(\mathbf{x}) = \mathbf{a}\mathbf{x}^2 + \mathbf{b}\mathbf{x} + \mathbf{c} \; ; \; \mathbf{a} \neq 0$$

Five Point Method

$$\left[\begin{array}{c}\text{MATH by Wilson}\\ \text{Your Personal Mathematics Trainer}\\ \text{MathByWilson.com}\end{array}\right]$$

Given

$$\mathbf{y} = \mathbf{f}(\mathbf{x}) = 4\mathbf{x}^2 + 8\mathbf{x} - 2$$
$$\mathbf{a} = 4$$
$$\mathbf{b} = 8$$
$$\mathbf{c} = -2$$

complete a **Five Point Method Template**:

a. **Domain**: Allowable x-values

$$\mathbf{Domain} = \mathbb{R}_{\mathbf{x}} = (-\infty, +\infty)_{\mathbf{x}}$$

b. **POINTS #1 & #2:** Max of two **x-intercept points**

Set $\mathbf{D} = \mathbf{b}^2 - 4\mathbf{ac}$

(1) $\mathbf{D} < 0$: NO x-intercept point
(2) $\mathbf{D} = 0$: ONE x-intercept point
(3) $\mathbf{D} > 0$: TWO x-intercept points

x-intercepts POINTS(s): $\left(\dfrac{-\mathbf{b} \pm \sqrt{\mathbf{b}^2 - 4\mathbf{ac}}}{2\mathbf{a}}, 0\right)$

$$\mathbf{x} = \frac{-\mathbf{b} \pm \sqrt{\mathbf{b}^2 - 4\mathbf{ac}}}{2\mathbf{a}} = \frac{-[8] \pm \sqrt{[8]^2 - 4[4][-2]}}{2[4]} = \frac{-8 \pm \sqrt{96}}{8}$$
$$= \frac{-8 \pm 4\sqrt{6}}{8} = -1 \pm \frac{\sqrt{6}}{2} \Rightarrow \mathbf{x} \approx -2.2247, 0.22474$$
$$\Rightarrow \left(-1 - \frac{\sqrt{6}}{2}, 0\right) \approx (-2.22, 0) \; ; \; \left(-1 + \frac{\sqrt{6}}{2}, 0\right) \approx (0.22)$$

c. **POINT #3:**
y-intercept POINT: $(0,\mathbf{c}) = (0,-2)$

d. **POINT #4:**
Symmetry POINT: $\left(-\frac{\mathbf{b}}{\mathbf{a}},\mathbf{c}\right)$ **;** also called "cheap point"

$$\left(-\frac{\mathbf{b}}{\mathbf{a}},\mathbf{c}\right) = \left(-\frac{[8]}{4},[-2]\right) = (-2,-2)$$

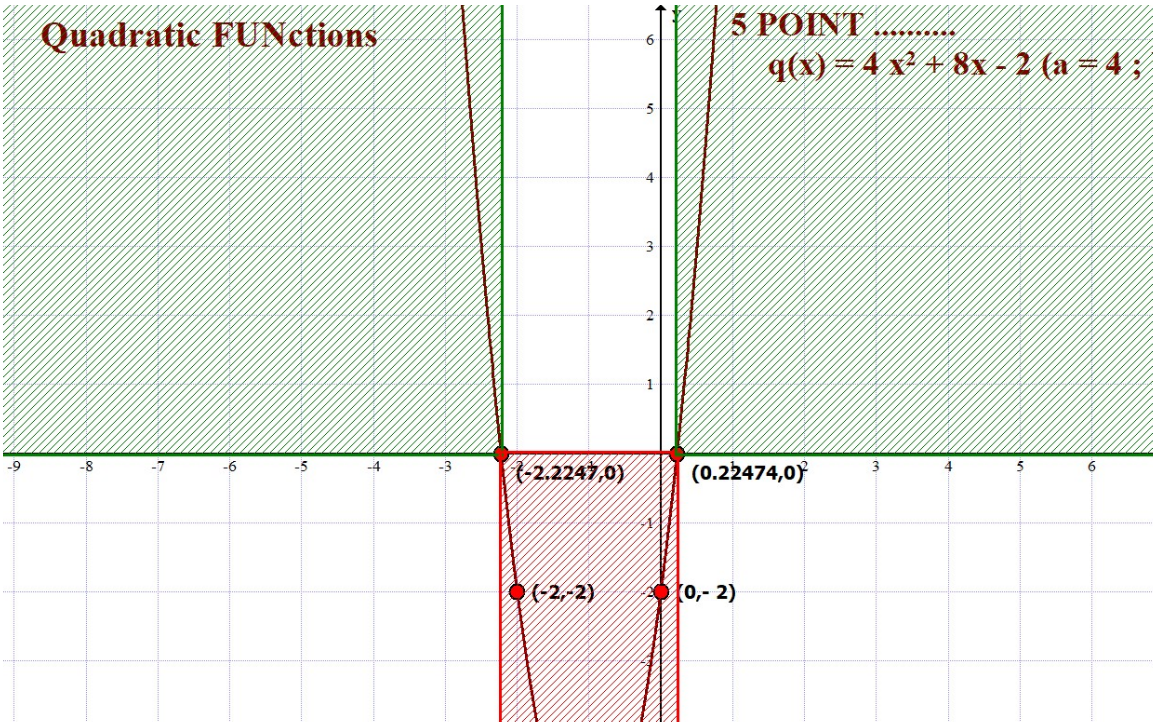
e. **POINT #5:**
Vertex POINT: $\left(-\frac{\mathbf{b}}{2\mathbf{a}},\frac{4\mathbf{ac}-\mathbf{b}^2}{4\mathbf{a}}\right)$

Absolute Maximum point if $\mathbf{a} < 0$
Absolute Minimum point if $\mathbf{a} > 0$

$\mathbf{a} = 4 > 0 \Rightarrow$ Minimum

$$\Rightarrow\left(-\frac{\mathbf{b}}{2\mathbf{a}},\frac{4\mathbf{ac}-\mathbf{b}^2}{4\mathbf{a}}\right) = \left(-\frac{[8]}{2[4]},\frac{4[4][-2]-[8]^2}{4[4]}\right) = (-1,-6)$$

f. **Graph:** PLOT the five (5) points (Max) and sketch the graph



g. **Positive:** The x-values where $\mathbf{y} > 0$

Negative: The x-values where $\mathbf{y} < 0$

Assuming two (2) x-intercept points, we have

(1) If $\mathbf{a} > 0$

$$\textbf{Positive} = \left(-\infty, \frac{-\mathbf{b} - \sqrt{\mathbf{b}^2 - 4\mathbf{ac}}}{2\mathbf{a}}\right] \cup \left[\frac{-\mathbf{b} + \sqrt{\mathbf{b}^2 - 4\mathbf{ac}}}{2\mathbf{a}}, +\infty\right)$$

$$\textbf{Negative} = \left[\frac{-\mathbf{b} - \sqrt{\mathbf{b}^2 - 4\mathbf{ac}}}{2\mathbf{a}}, \frac{-\mathbf{b} + \sqrt{\mathbf{b}^2 - 4\mathbf{ac}}}{2\mathbf{a}}\right]$$

$$\textbf{Neg f} = (-2.22, 0.22)_{\mathbf{x}}$$
$$\textbf{Pos f} = (-\infty, -2.22)_{\mathbf{x}} \cup (0.22, +\infty)_{\mathbf{x}}$$

(2) If $\mathbf{a} < 0$

$$\textbf{Positive} = \left[\frac{-\mathbf{b} - \sqrt{\mathbf{b}^2 - 4\mathbf{ac}}}{2\mathbf{a}}, \frac{-\mathbf{b} + \sqrt{\mathbf{b}^2 - 4\mathbf{ac}}}{2\mathbf{a}}\right]$$

$$\textbf{Negative} = \left(-\infty, \frac{-\mathbf{b} - \sqrt{\mathbf{b}^2 - 4\mathbf{ac}}}{2\mathbf{a}}\right] \cup \left[\frac{-\mathbf{b} + \sqrt{\mathbf{b}^2 - 4\mathbf{ac}}}{2\mathbf{a}}, +\infty\right)$$

h. **Range:** Allowable y-values

$$\textbf{Range} = \left(-\infty, \frac{4\mathbf{ac} - \mathbf{b}^2}{4\mathbf{a}}\right]_{\mathbf{y}} \text{ if } \mathbf{a} < 0$$

$$\textbf{Range} = \left[\frac{4\mathbf{ac} - \mathbf{b}^2}{4\mathbf{a}}, +\infty\right)_{\mathbf{y}} \text{ if } \mathbf{a} > 0$$

$$[-6, +\infty)_{\mathbf{y}}$$

